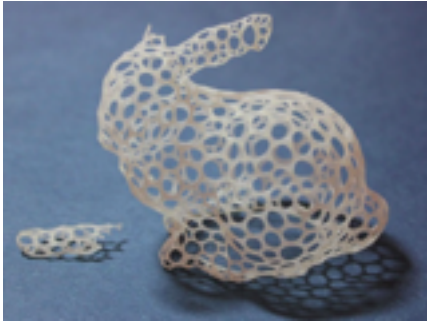
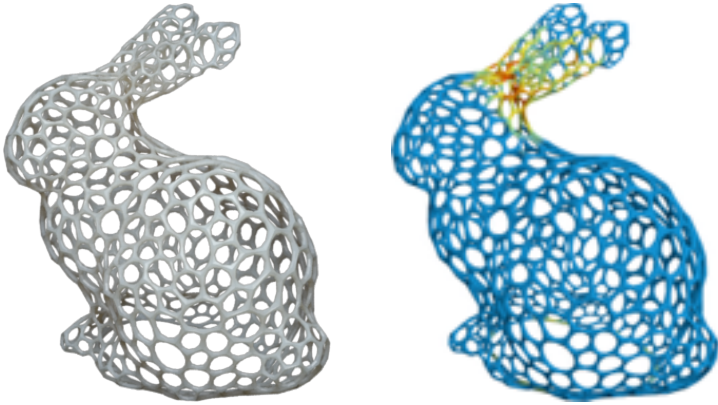


Mesh Arrangements for Solid Geometry

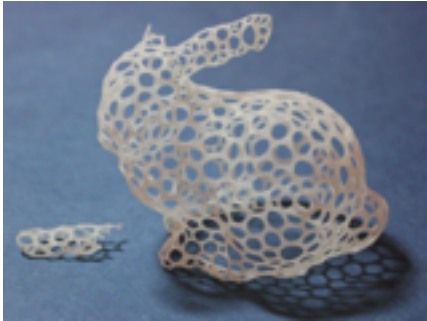
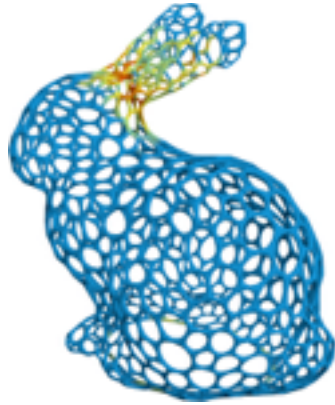
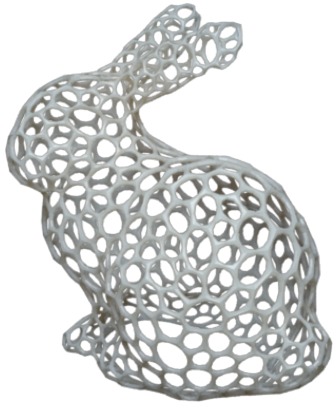
Qingnan Zhou, Eitan Grinspun, Denis Zorin, Alec Jacobson

Motivation: Fabrication

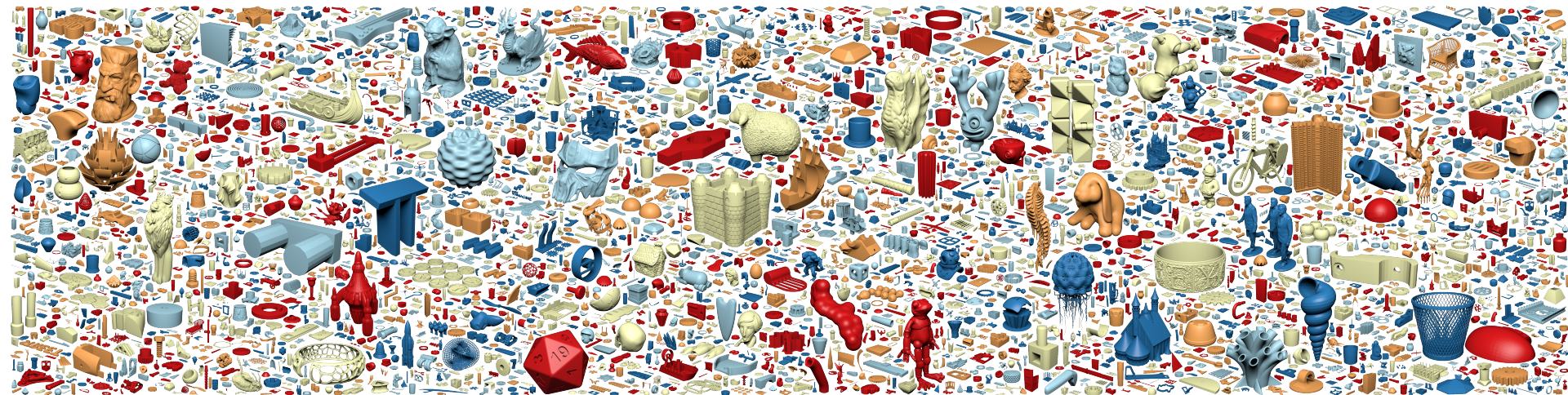
Motivation: Fabrication



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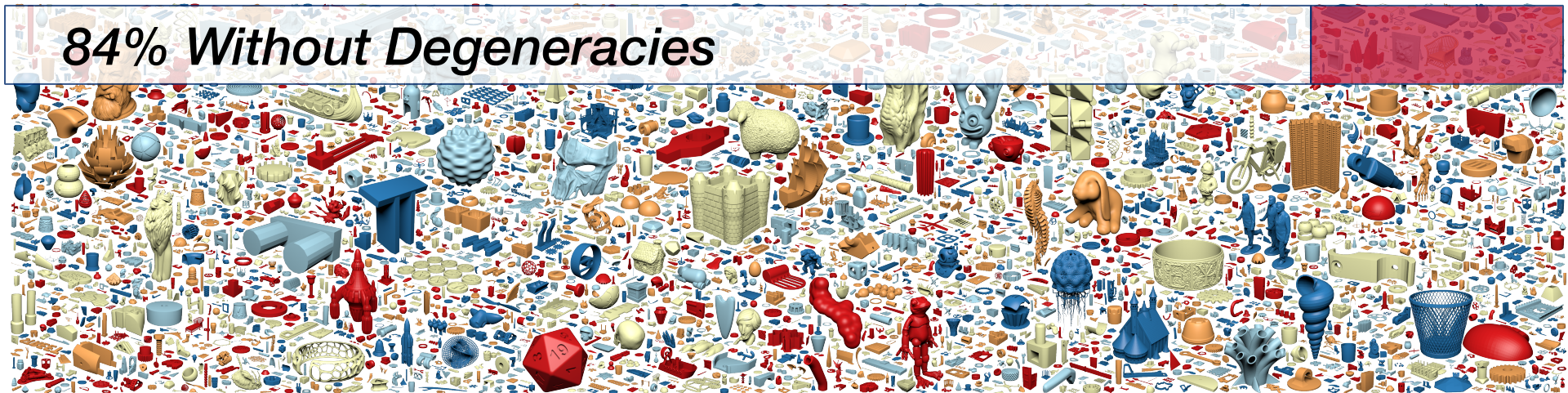


Thingi10K Dataset



Thingi10K Dataset

84% Without Degeneracies



Thingi10K Dataset

84% Without Degeneracies

78% Manifold

74% Single Component



Thingi10K Dataset

84% Without Degeneracies

78% Manifold

74% Single Component

69% Without Coplanar Intersections

Thingi10K Dataset

84% Without Degeneracies

78% Manifold

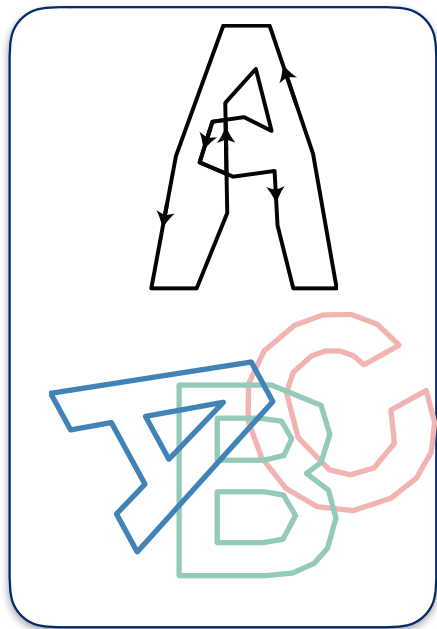
74% Single Component

69% Without Coplanar Intersections

55% Without self-intersections

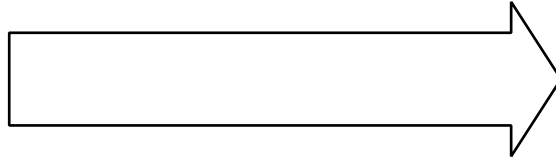
Problem Statement

Input

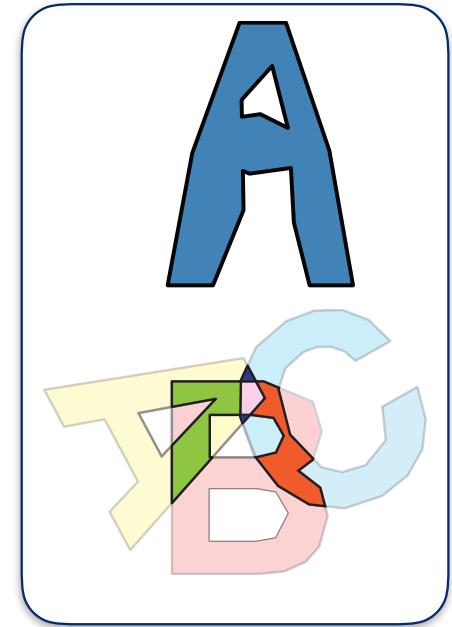


Problem Statement

Input



Output Solid

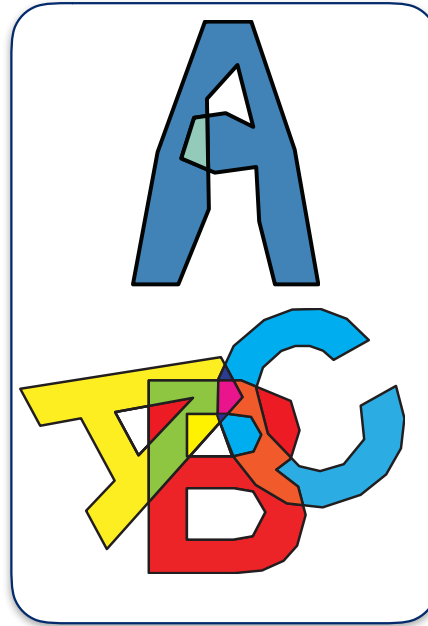


Problem Statement

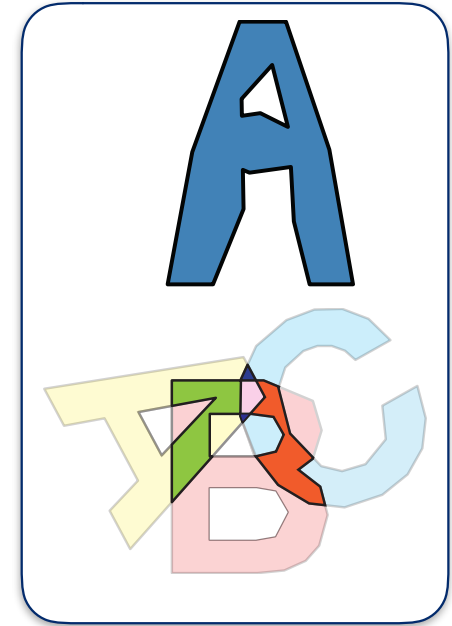
Input



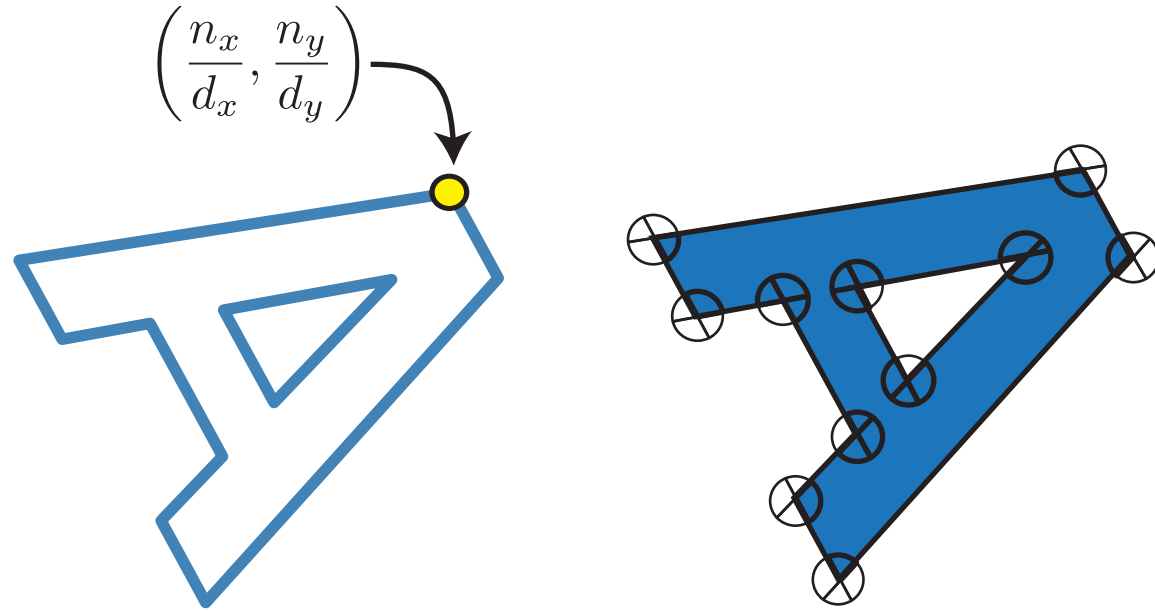
Mesh Arrangement



Output Solid

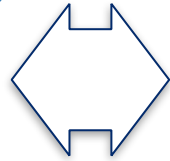
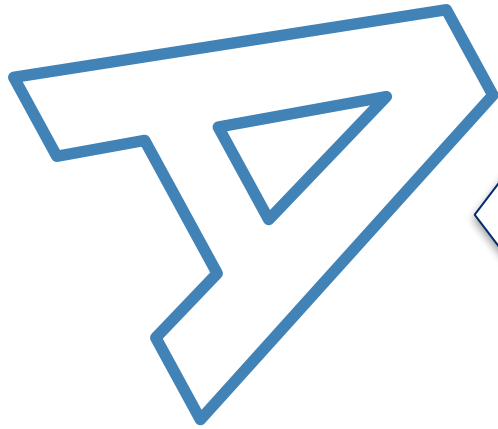


Previous Works: CGAL

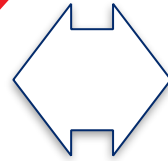
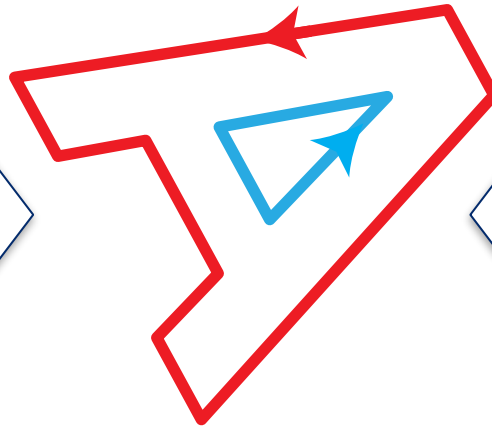


Previous Works: CGAL

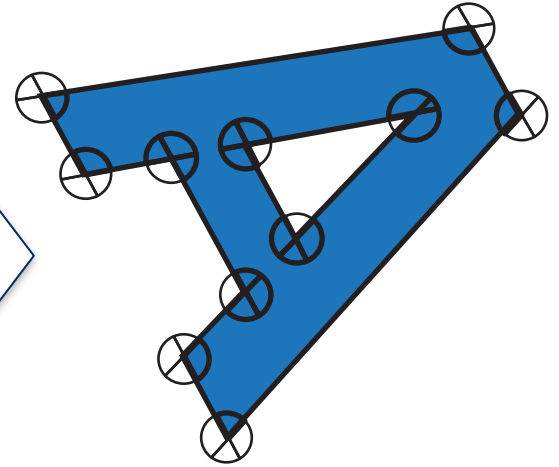
Input Mesh



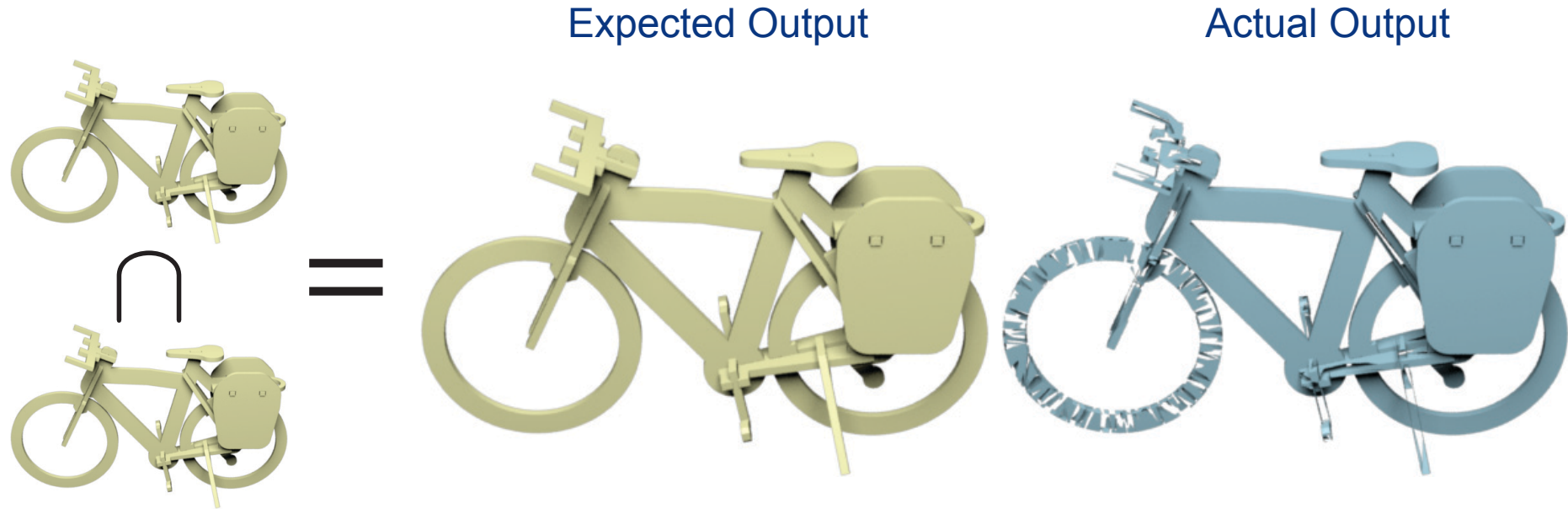
Polyhedron



Nef Polyhedron

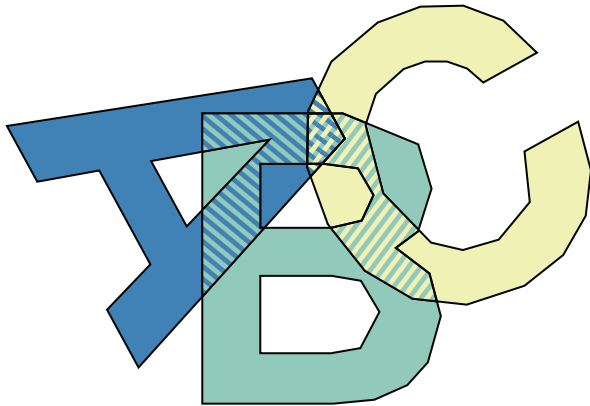


Previous Works: Cork, QuickCSG

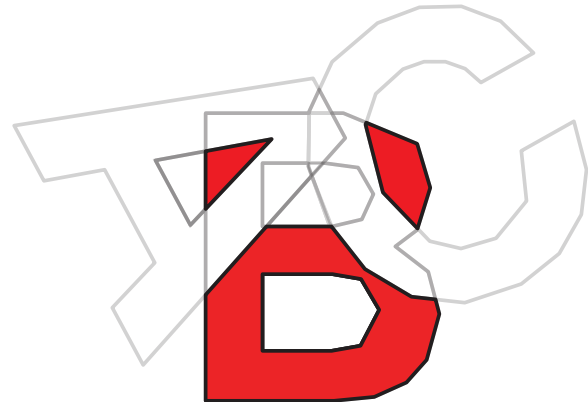


Input and Output

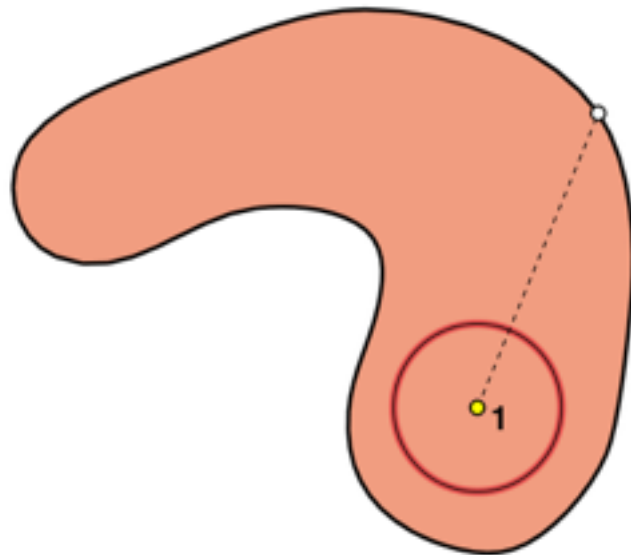
Valid Input



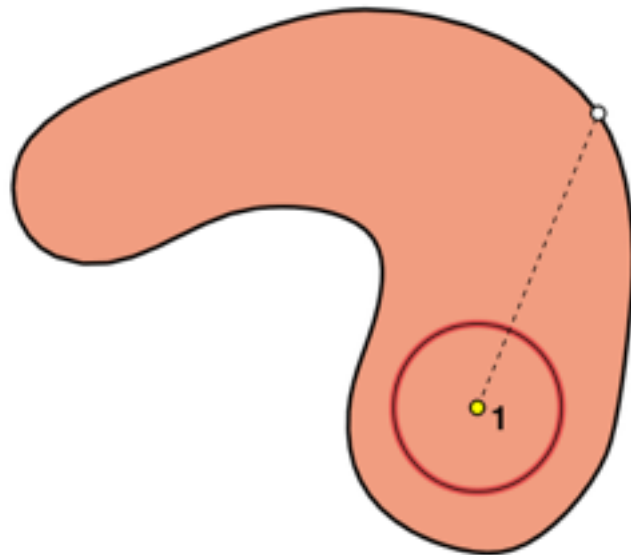
Valid Output



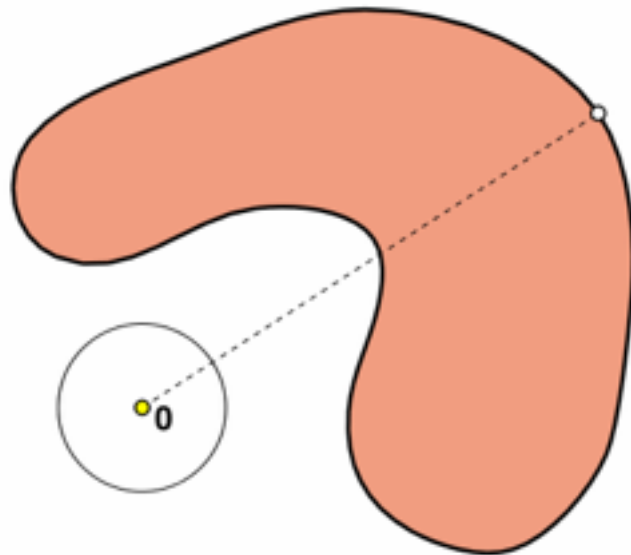
Winding Number



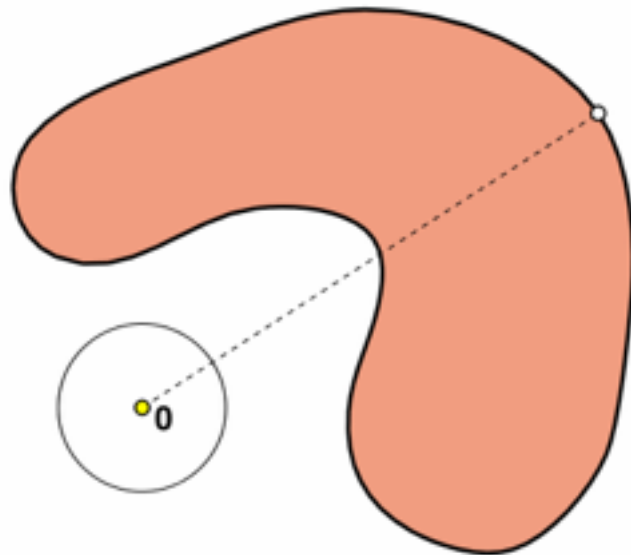
Winding Number



Winding Number

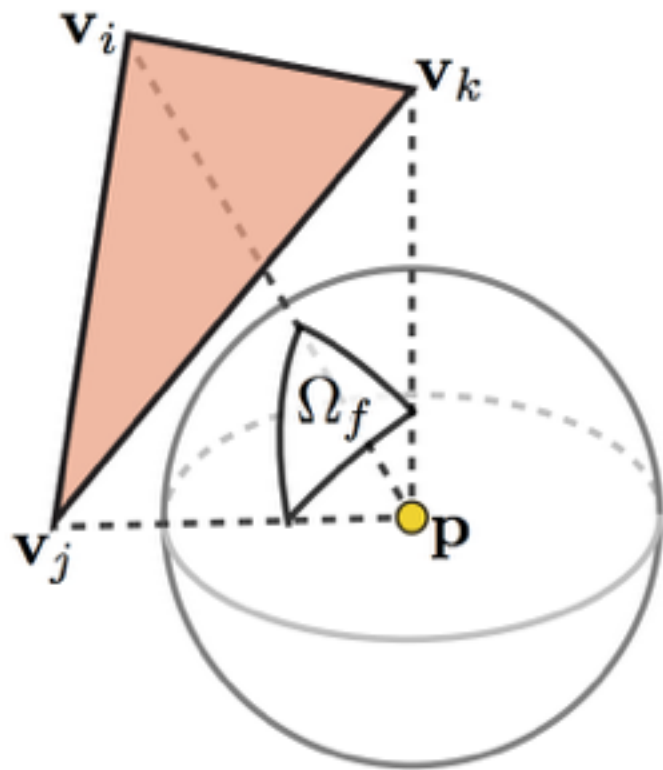


Winding Number

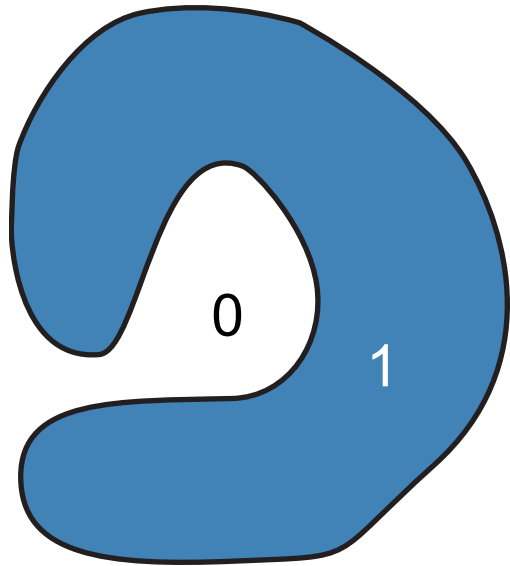


Winding Number

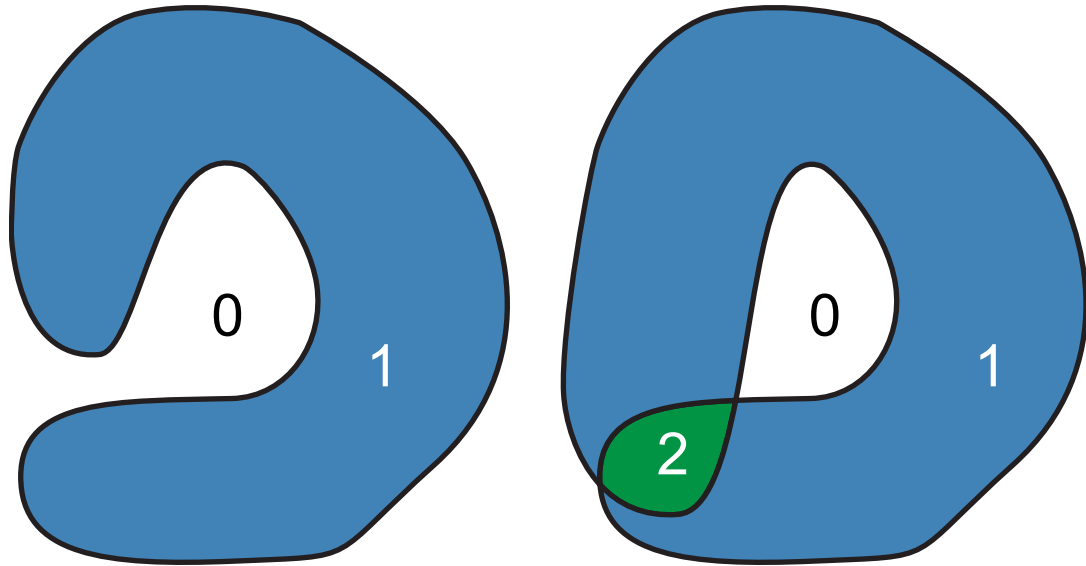
$$w(p) = \frac{1}{4\pi} \sum_{f=1}^m \Omega_f(p)$$



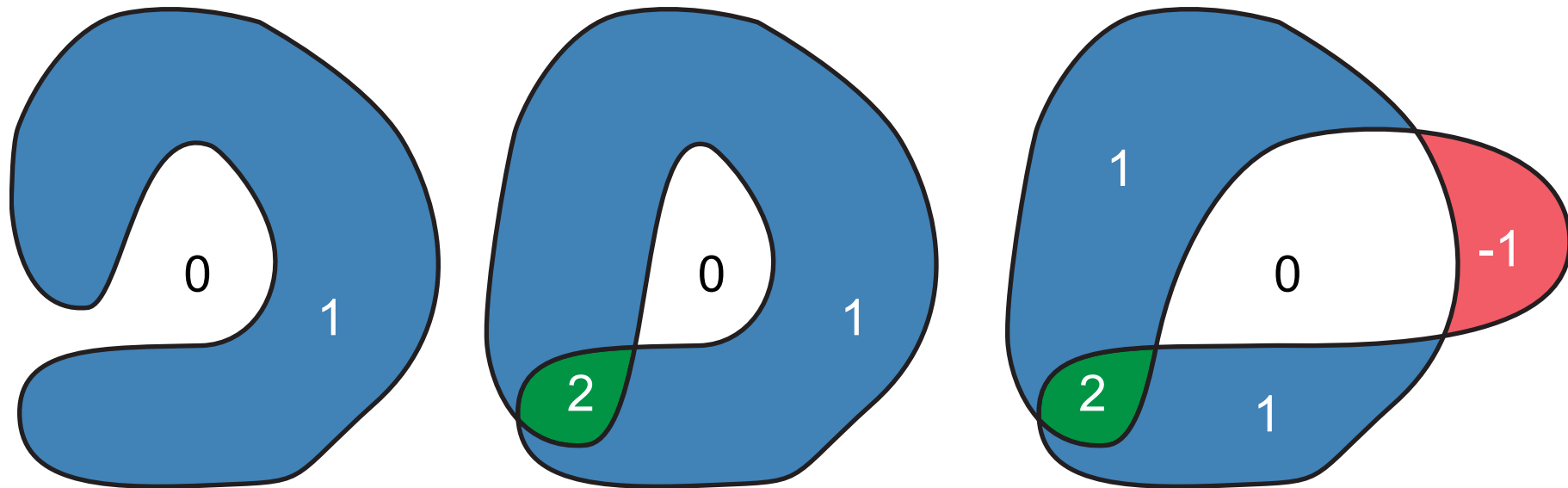
Winding Number



Winding Number



Winding Number



Definitions

PWN Mesh:

A mesh that induces **P**iecewise-constant **W**inding **N**umbers.

Definitions

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A mesh that induces **P**iecewise-constant **W**inding **N**umbers.

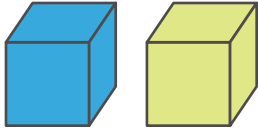
Solid Mesh:

A PWN mesh that

- induces 0/1 winding number field
- without self-intersection or degeneracies

PWN Mesh

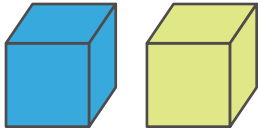
PWN Mesh: A mesh that induces **P**iecewise-constant **W**inding **N**umbers.



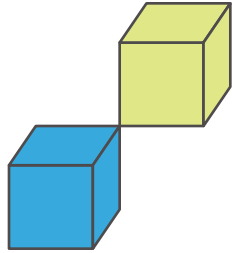
Multiple Components

PWN Mesh

PWN Mesh: A mesh that induces **P**iecewise-constant **W**inding **N**umbers.



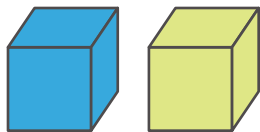
Multiple Components



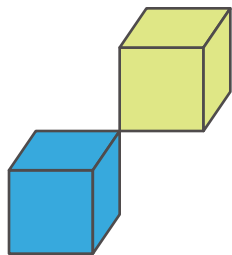
Vertex Non-manifold

PWN Mesh

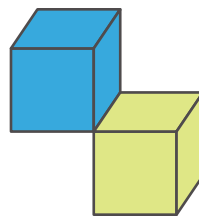
PWN Mesh: A mesh that induces **P**iecewise-constant **W**inding **N**umbers.



Multiple Components



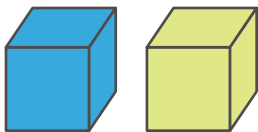
Vertex Non-manifold



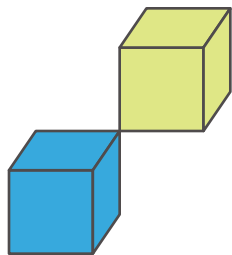
Edge Non-manifold

PWN Mesh

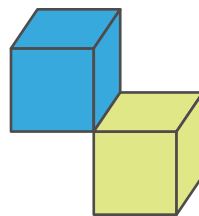
PWN Mesh: A mesh that induces **P**iecewise-constant **W**inding **N**umbers.



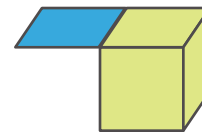
Multiple Components



Vertex Non-manifold



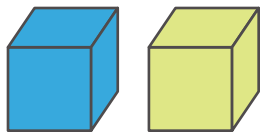
Edge Non-manifold



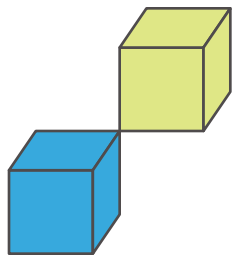
Degenerated Faces

PWN Mesh

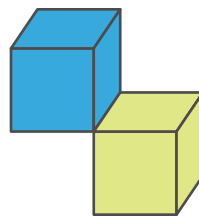
PWN Mesh: A mesh that induces **P**iecewise-constant **W**inding **N**umbers.



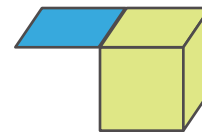
Multiple Components



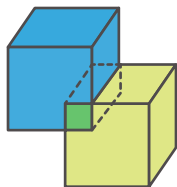
Vertex Non-manifold



Edge Non-manifold



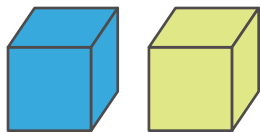
Degenerated Faces



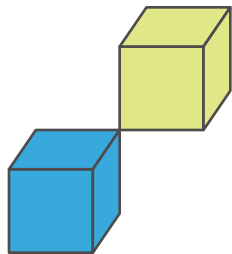
Self-intersection

PWN Mesh

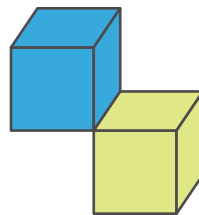
PWN Mesh: A mesh that induces **P**iecewise-constant **W**inding **N**umbers.



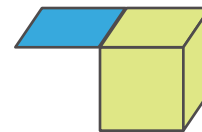
Multiple Components



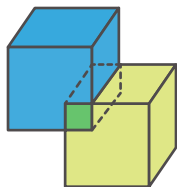
Vertex Non-manifold



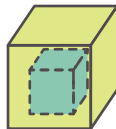
Edge Non-manifold



Degenerated Faces



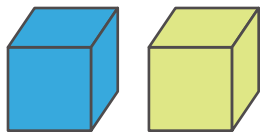
Self-intersection



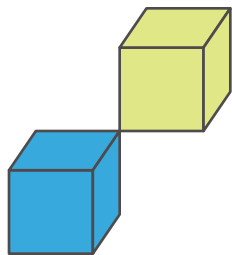
Nested Components

PWN Mesh

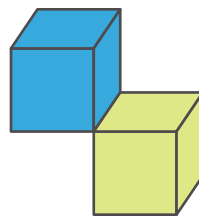
PWN Mesh: A mesh that induces **P**iecewise-constant **W**inding **N**umbers.



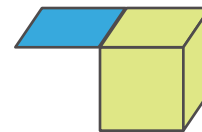
Multiple Components



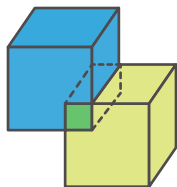
Vertex Non-manifold



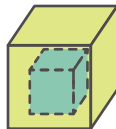
Edge Non-manifold



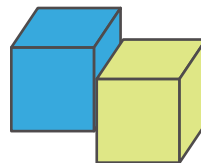
Degenerated Faces



Self-intersection



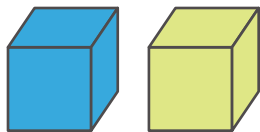
Nested Components



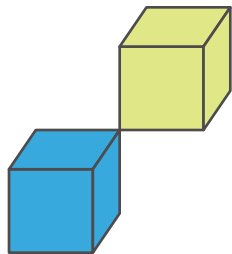
Overlapping Faces

PWN Mesh

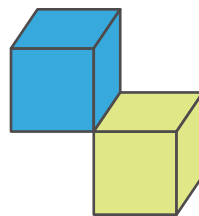
PWN Mesh: A mesh that induces **P**iecewise-constant **W**inding **N**umbers.



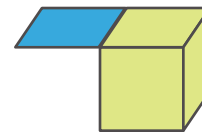
Multiple Components



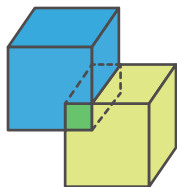
Vertex Non-manifold



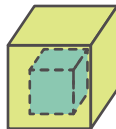
Edge Non-manifold



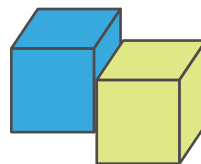
Degenerated Faces



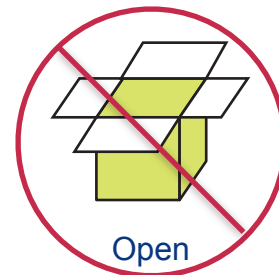
Self-intersection



Nested Components



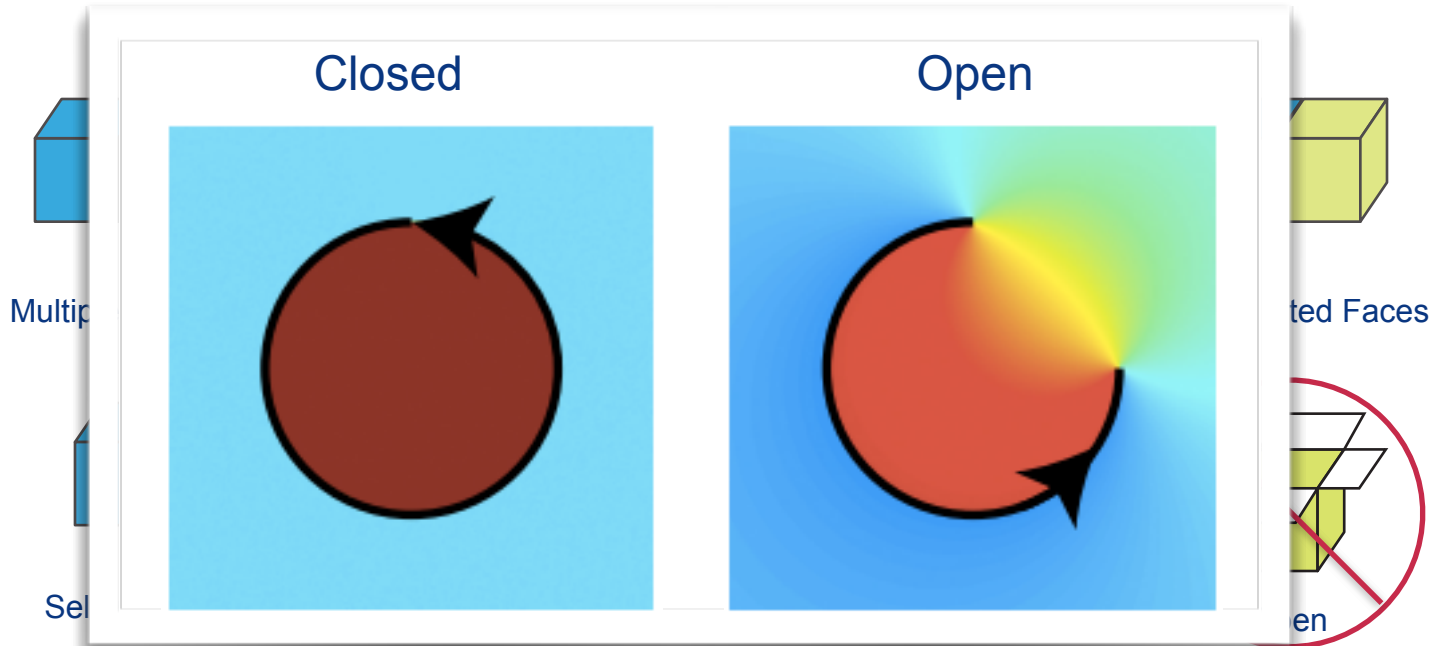
Overlapping Faces



Open

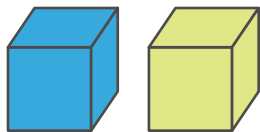
PWN Mesh

PWN Mesh: A mesh that induces **P**iecewise-constant **W**inding **N**umbers.

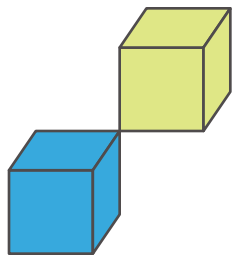


PWN Mesh

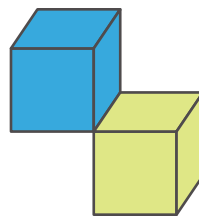
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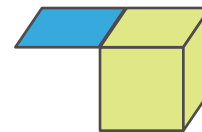
Multiple Components



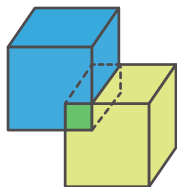
Vertex Non-manifold



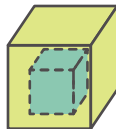
Edge Non-manifold



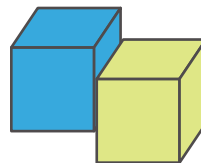
Degenerated Faces



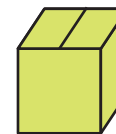
Self-intersection



Nested Components



Overlapping Faces



Topologically Open

Valid Input

10,000 Meshes from Thingiverse

86% PWN Meshes

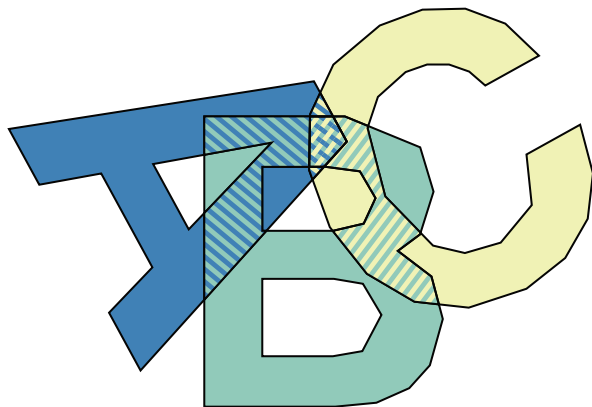
A large collection of 3D meshes from Thingiverse, with 86% highlighted in red to indicate they are PWN meshes. The meshes are arranged in a grid-like pattern, showing a variety of objects including tools, furniture, and decorative items. The red highlighting covers the right side of the image, while the left side is white.

51% Solid

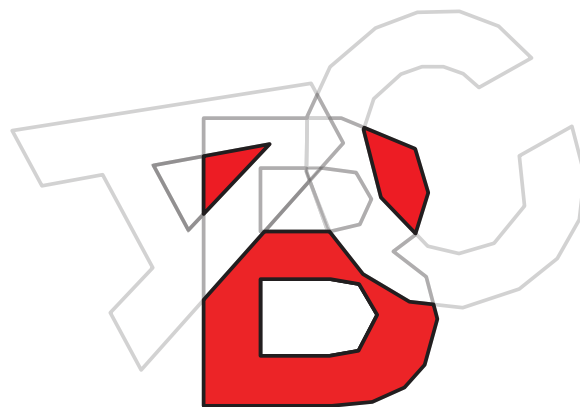
50% Polyhedron

Input and Output

Input: PWN Meshes

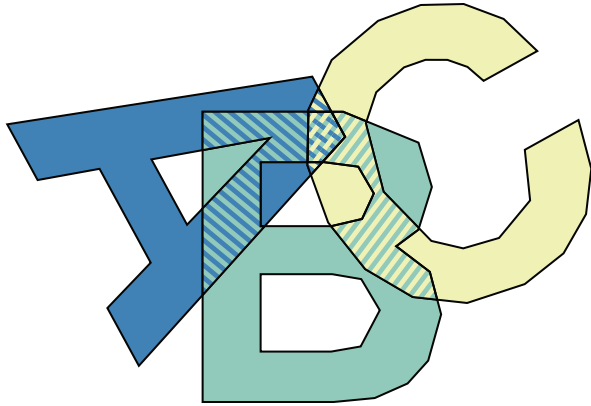


Output: Solid Mesh

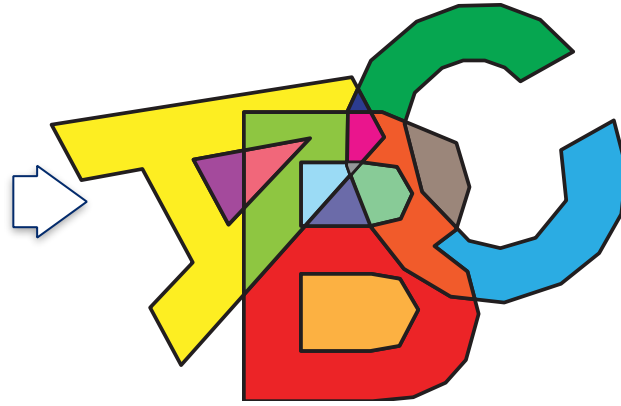


Input and Output

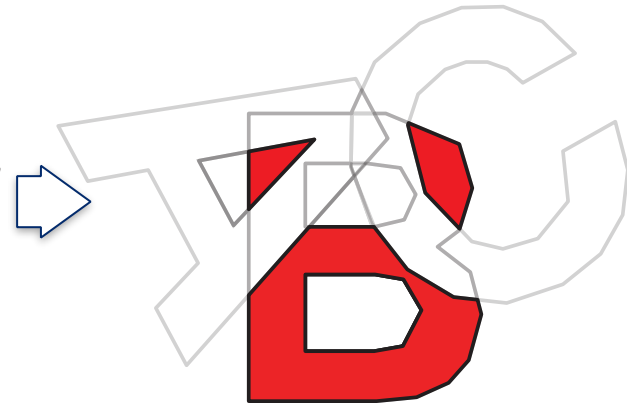
Input: PWN Meshes



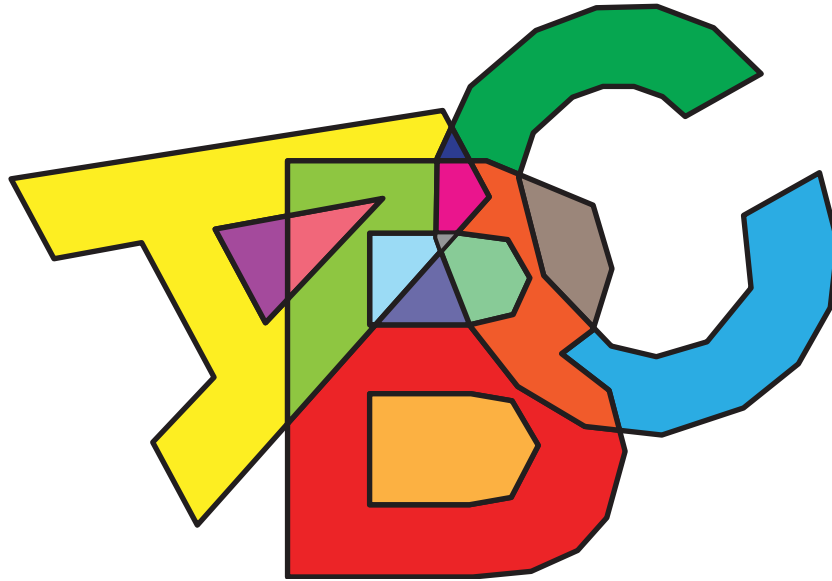
Mesh Arrangement



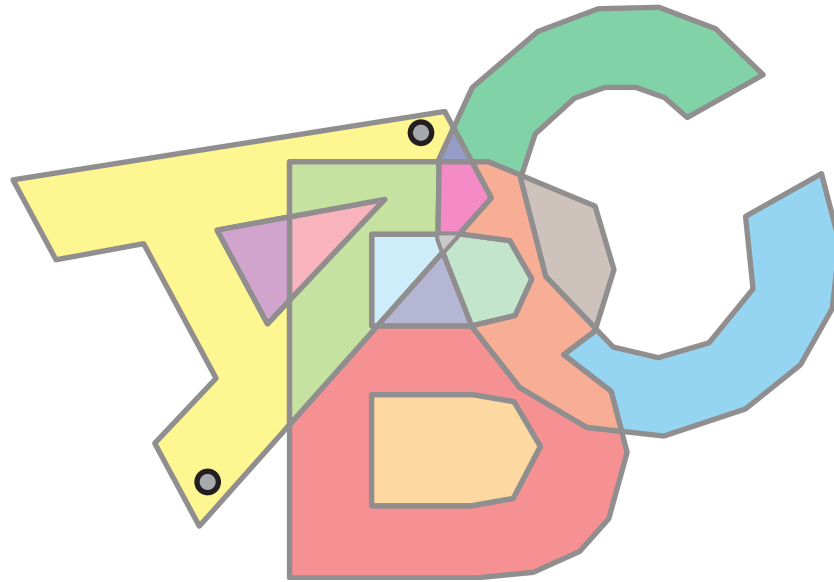
Output: Solid Mesh



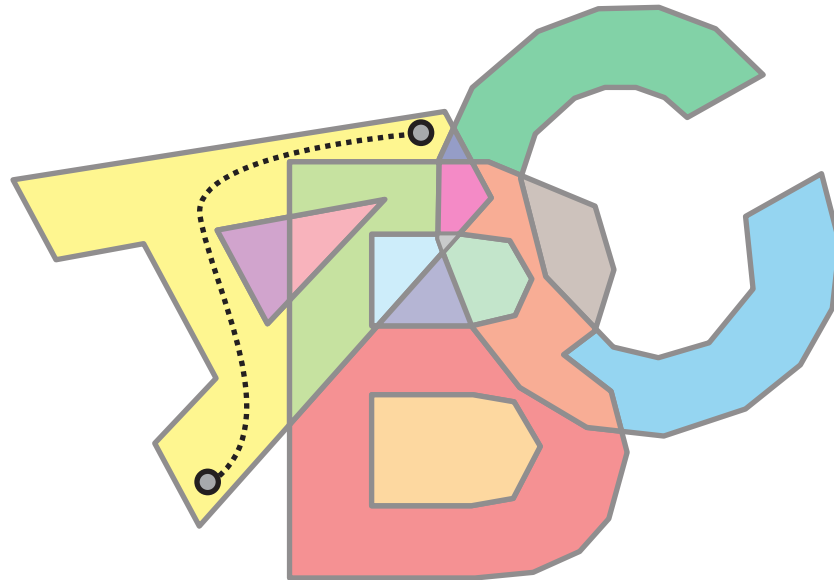
Mesh Arrangement



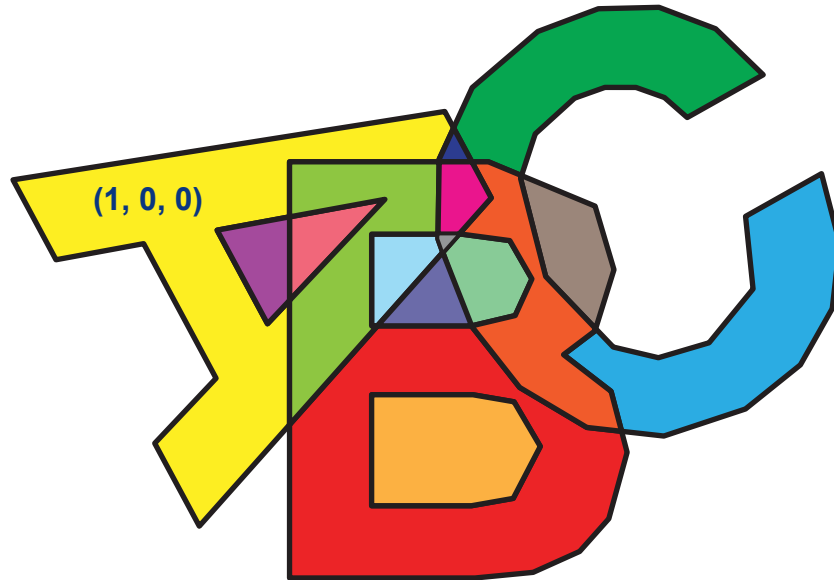
Mesh Arrangement



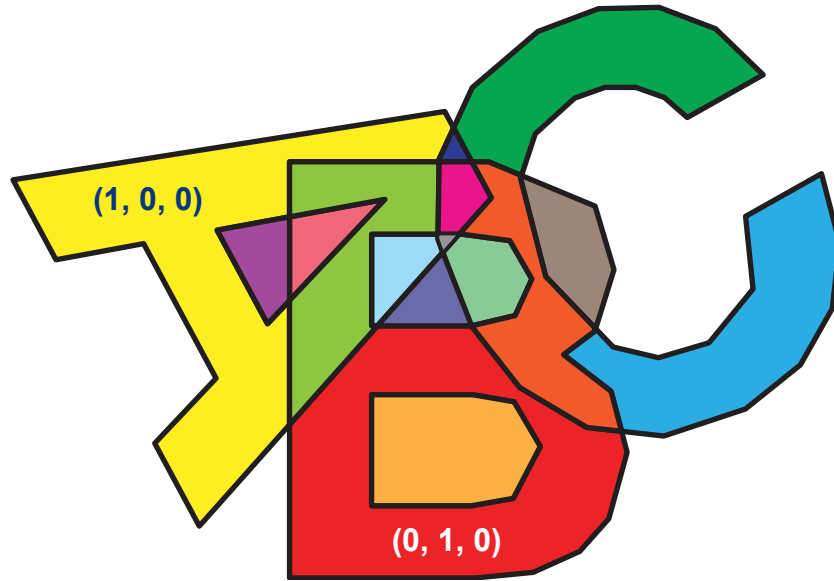
Mesh Arrangement



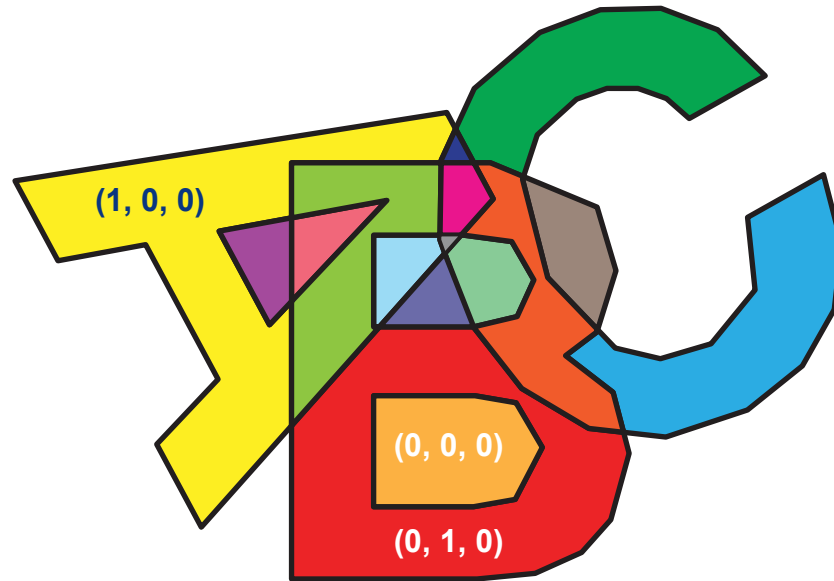
Mesh Arrangement



Mesh Arrangement



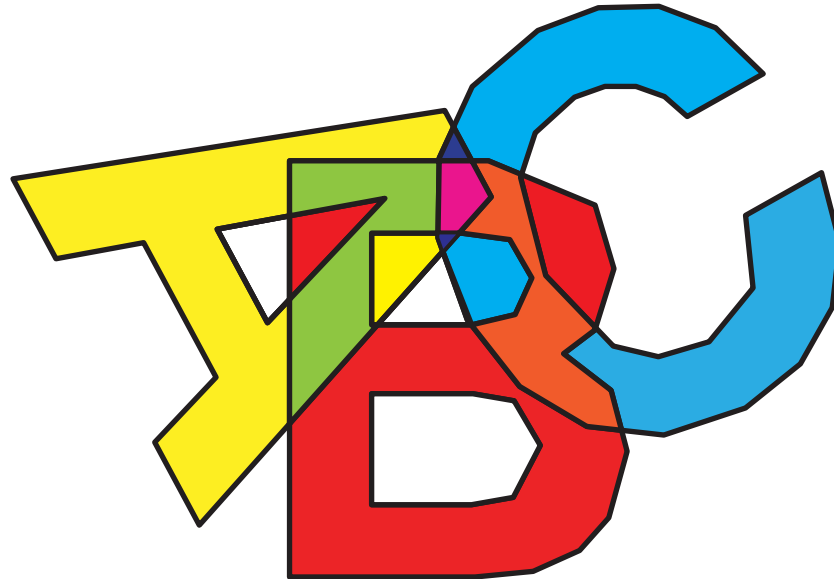
Mesh Arrangement



Mesh Arrangement

(w_A, w_B, w_C)

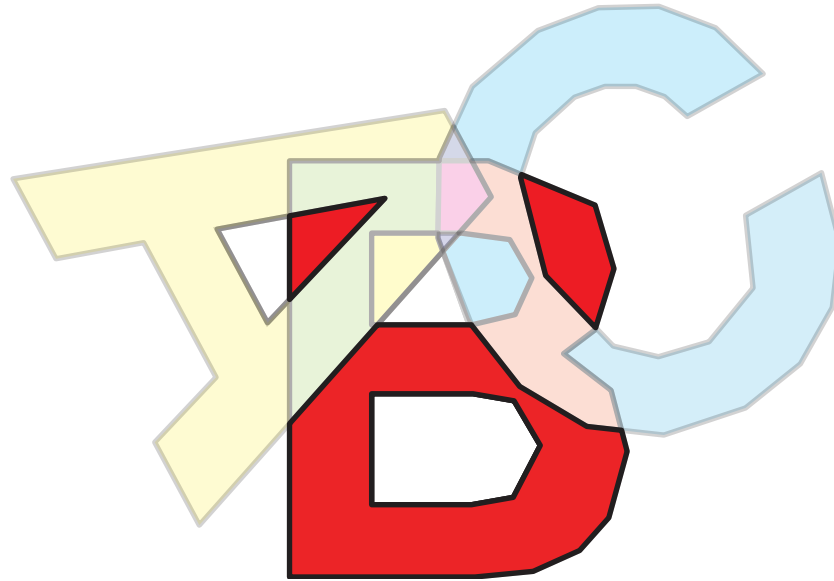
(0, 0, 0)
(1, 0, 0)
(0, 1, 0)
(0, 0, 1)
(1, 1, 0)
(0, 1, 1)
(1, 0, 1)
(1, 1, 1)



Mesh Arrangement

(w_A, w_B, w_C)

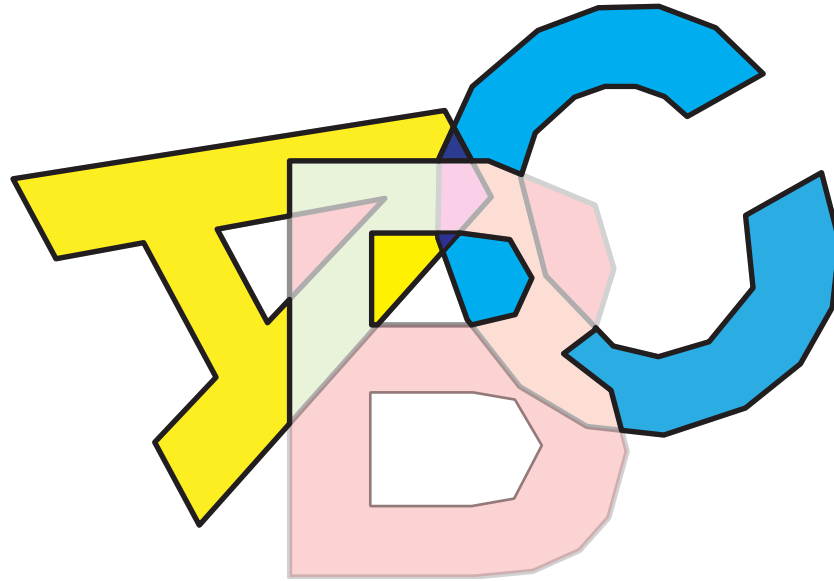
(0, 0, 0)
(1, 0, 0)
(0, 1, 0)
(0, 0, 1)
(1, 1, 0)
(0, 1, 1)
(1, 0, 1)
(1, 1, 1)



Mesh Arrangement

(w_A, w_B, w_C)

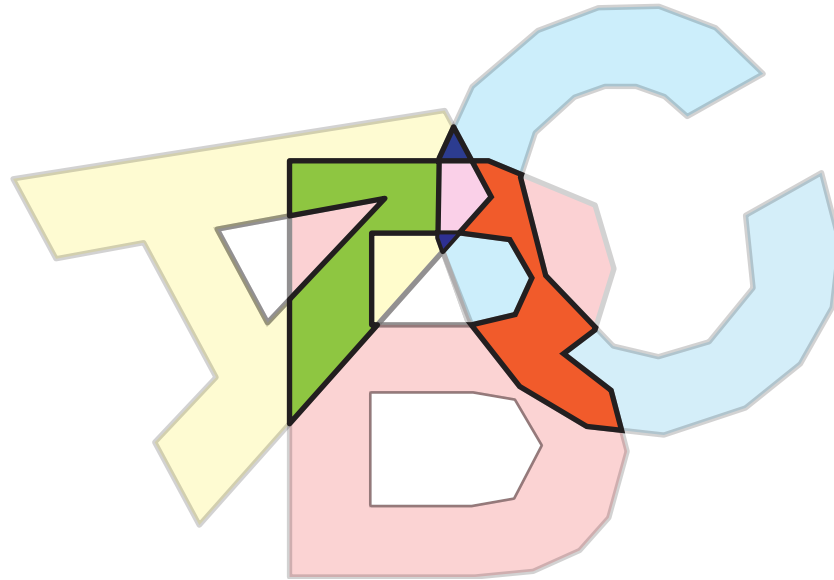
(0, 0, 0)
(1, 0, 0)
(0, 1, 0)
(0, 0, 1)
(1, 1, 0)
(0, 1, 1)
(1, 0, 1)
(1, 1, 1)



Mesh Arrangement

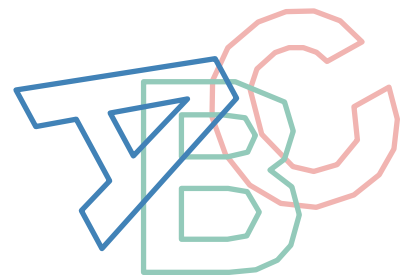
(w_A, w_B, w_C)

(0, 0, 0)
(1, 0, 0)
(0, 1, 0)
(0, 0, 1)
(1, 1, 0)
(0, 1, 1)
(1, 0, 1)
(1, 1, 1)



Algorithm

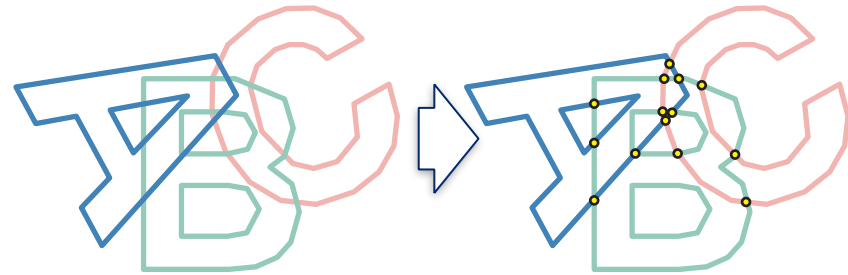
Input Meshes



Algorithm

Input Meshes

Resolving
(Self-)Intersections

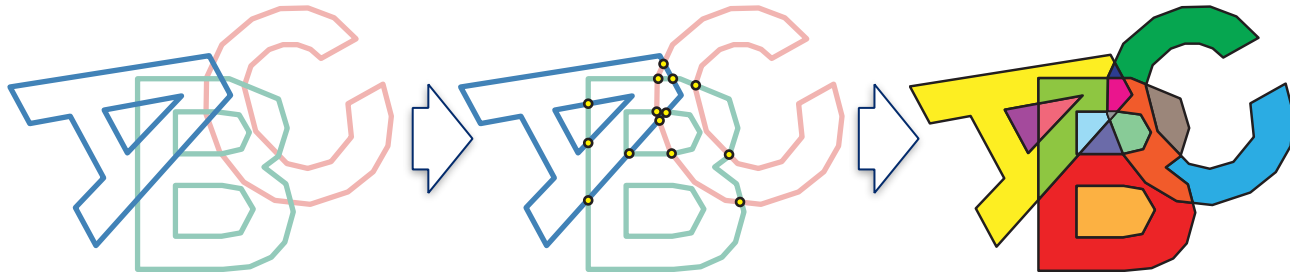


Algorithm

Input Meshes

Resolving
(Self-)Intersections

Extracting
Cells



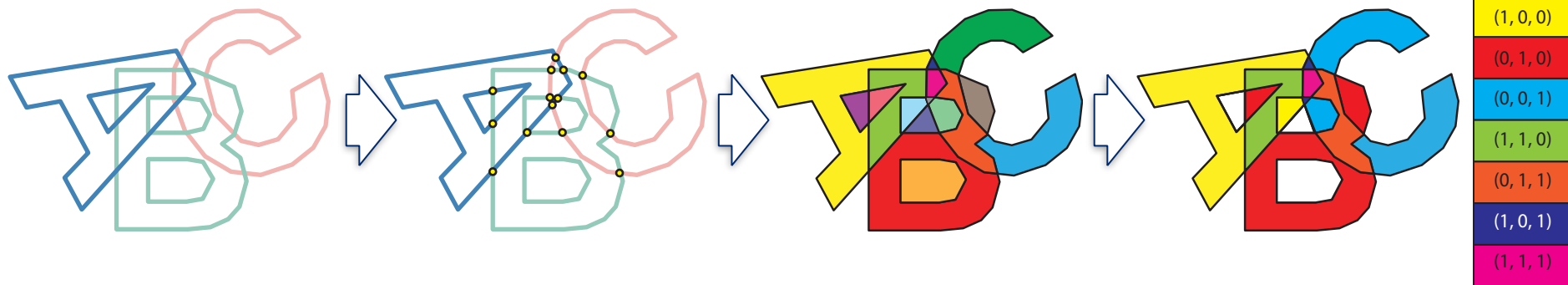
Algorithm

Input Meshes

Resolving
(Self-)Intersections

Extracting
Cells

Propagating
Winding Numbers

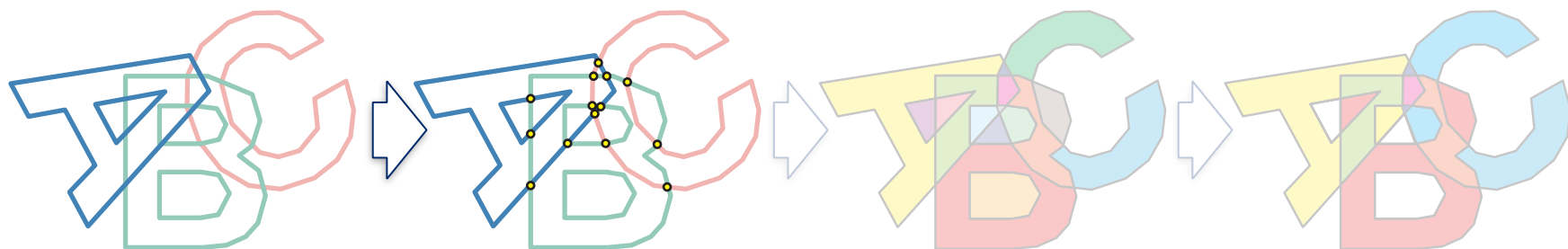


Algorithm

Resolving
(Self-)Intersections

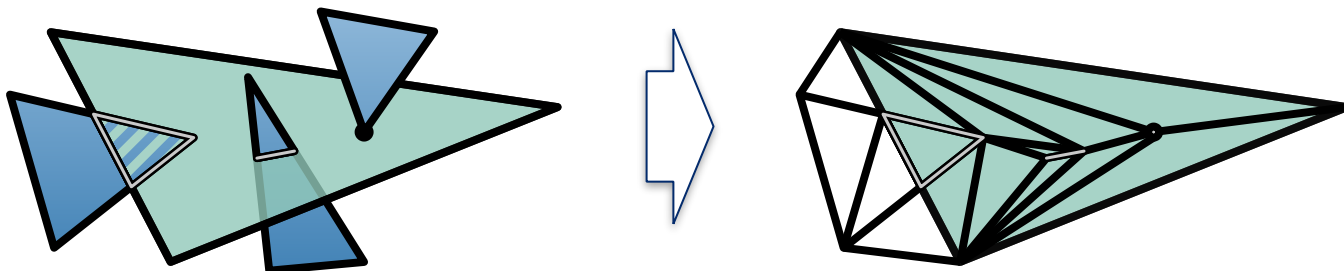
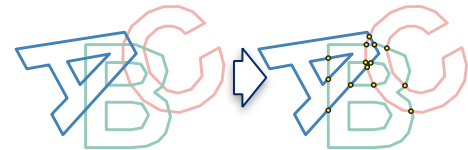
Extracting
Cells

Propagating
Winding Numbers

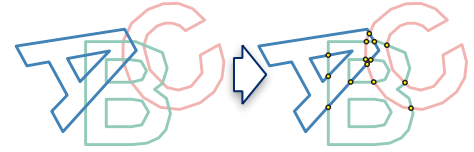


(0, 0, 0)
(1, 0, 0)
(0, 1, 0)
(0, 0, 1)
(1, 1, 0)
(0, 1, 1)
(1, 0, 1)
(1, 1, 1)

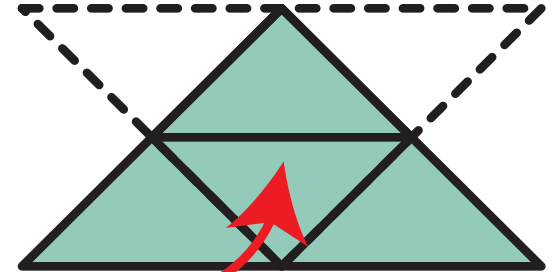
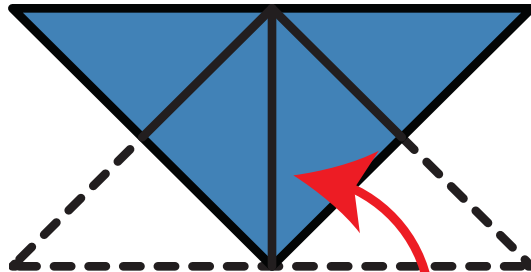
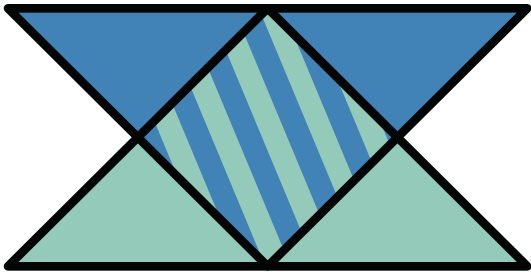
Resolving (Self-)Intersections



Resolving (Self-)Intersections



Pitfall: CDT is not unique, overlapping triangle needs to be consistently resolved!



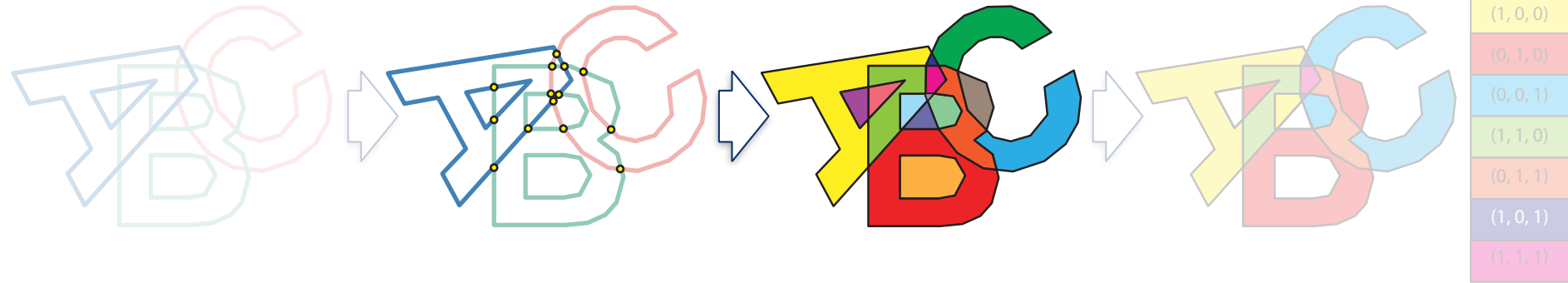
Inconsistent!

Algorithm

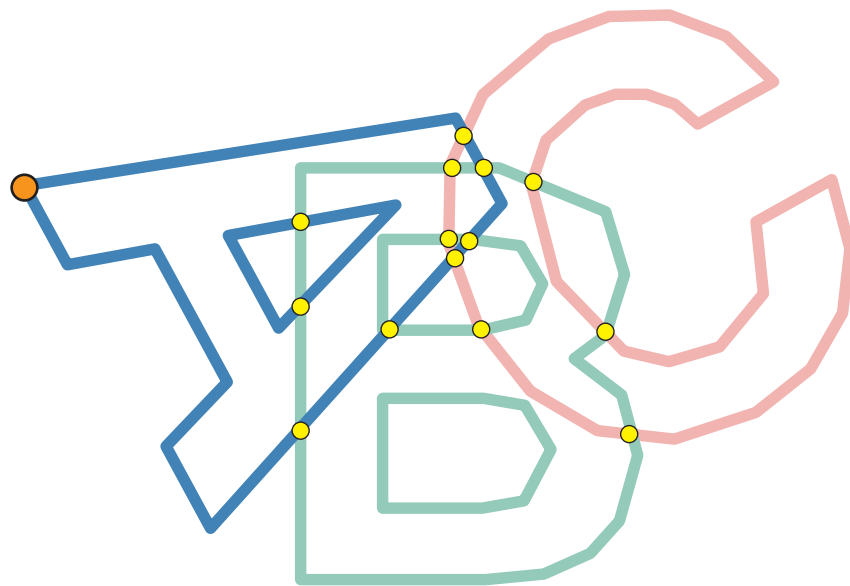
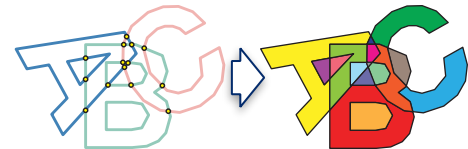
Resolving
(Self-)Intersections

Extracting
Cells

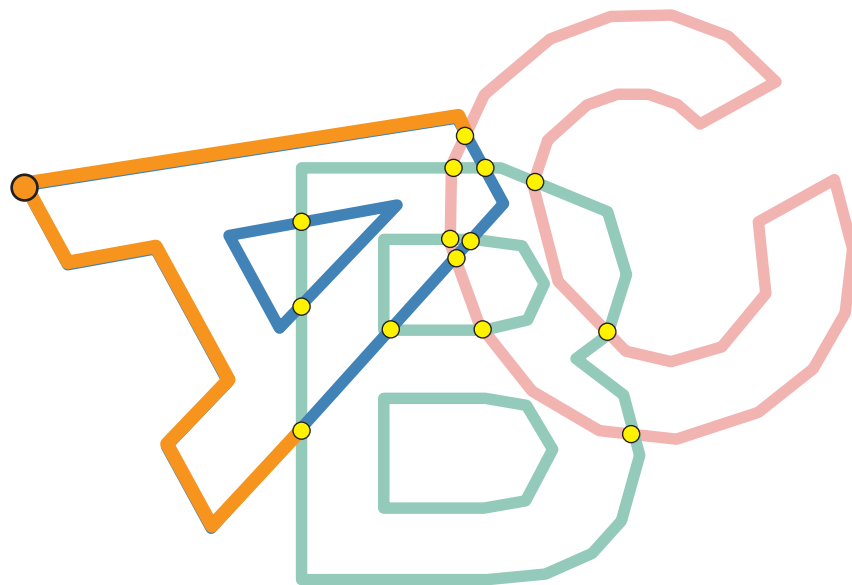
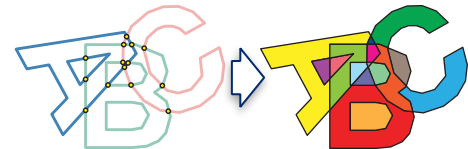
Propagating
Winding Numbers



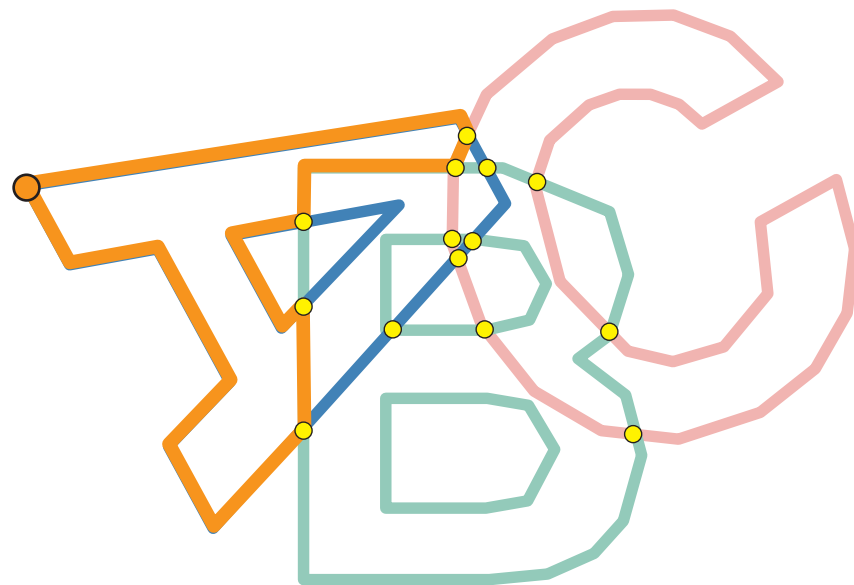
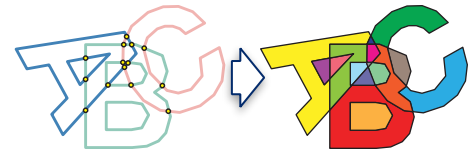
Extracting Cells



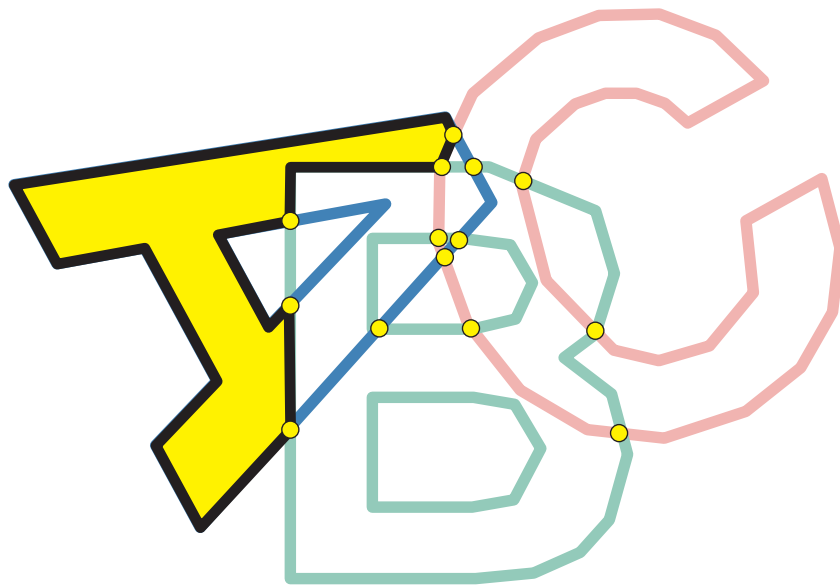
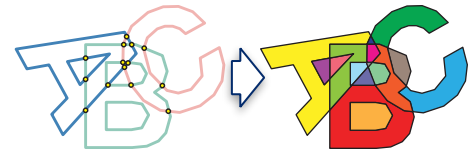
Extracting Cells



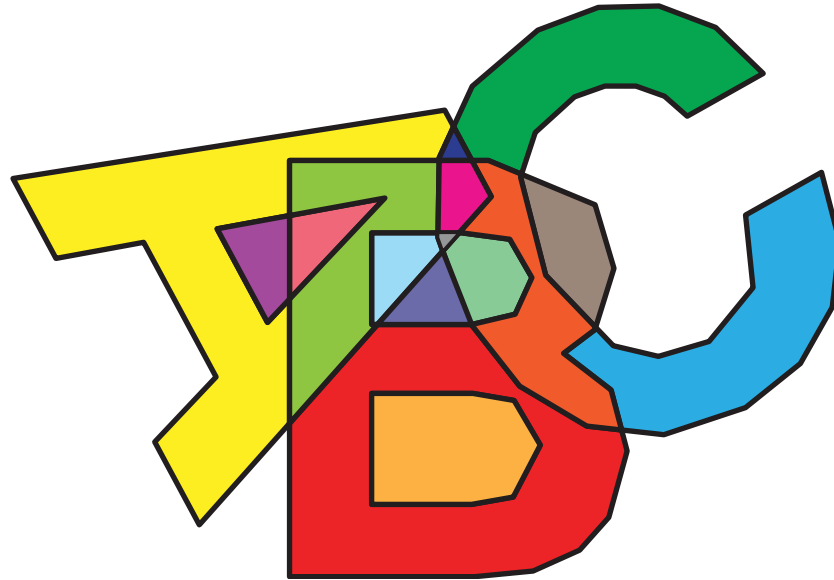
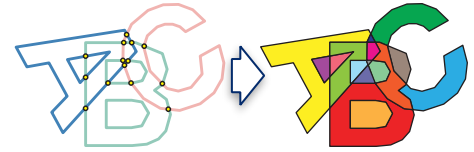
Extracting Cells



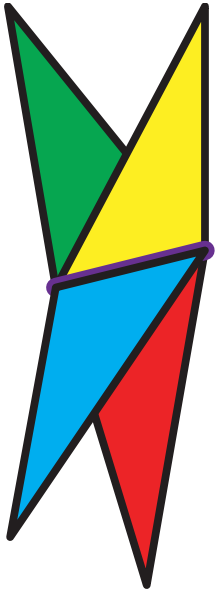
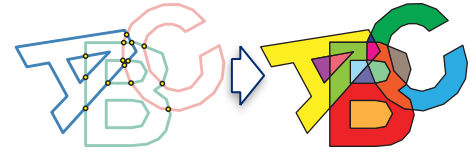
Extracting Cells



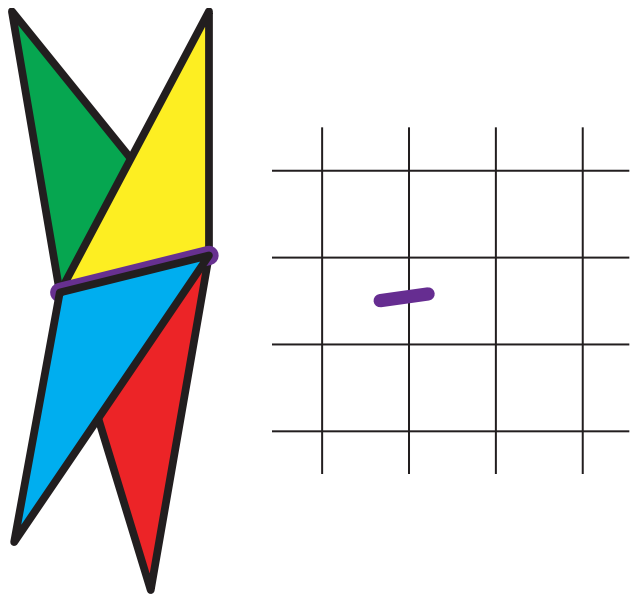
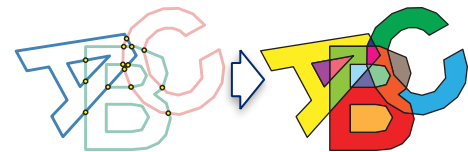
Extracting Cells



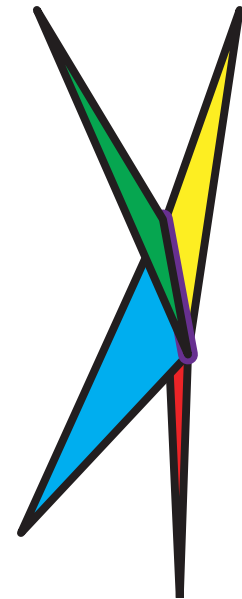
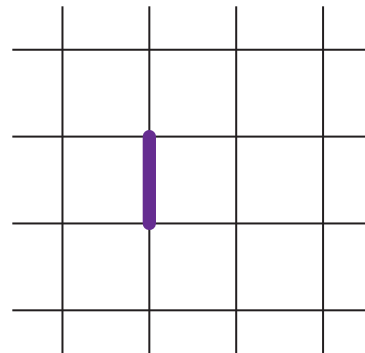
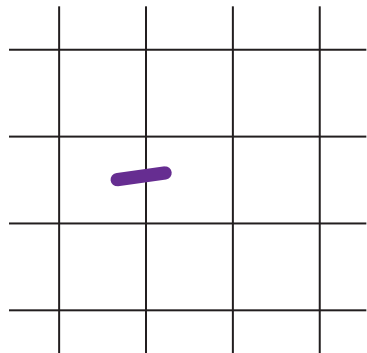
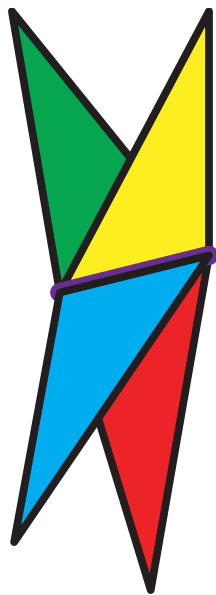
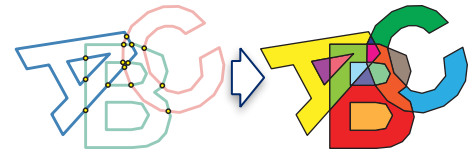
Extracting Cells



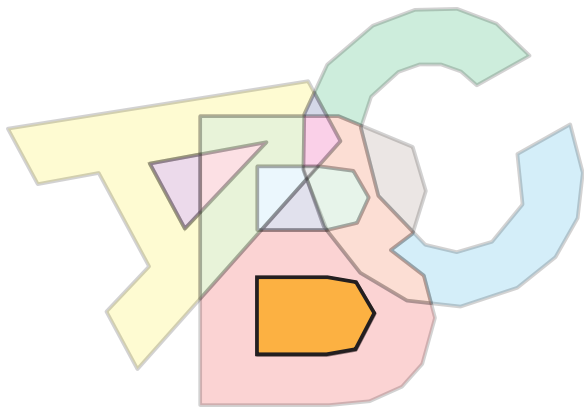
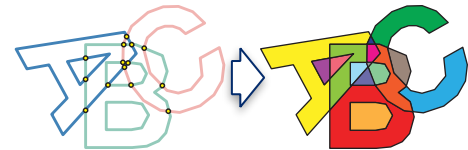
Extracting Cells



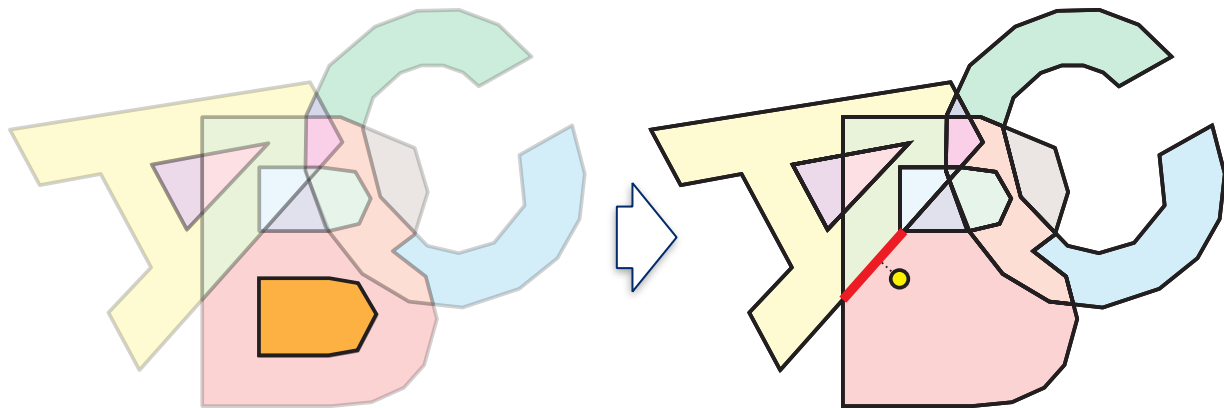
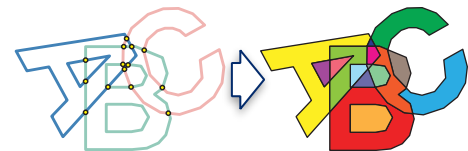
Extracting Cells



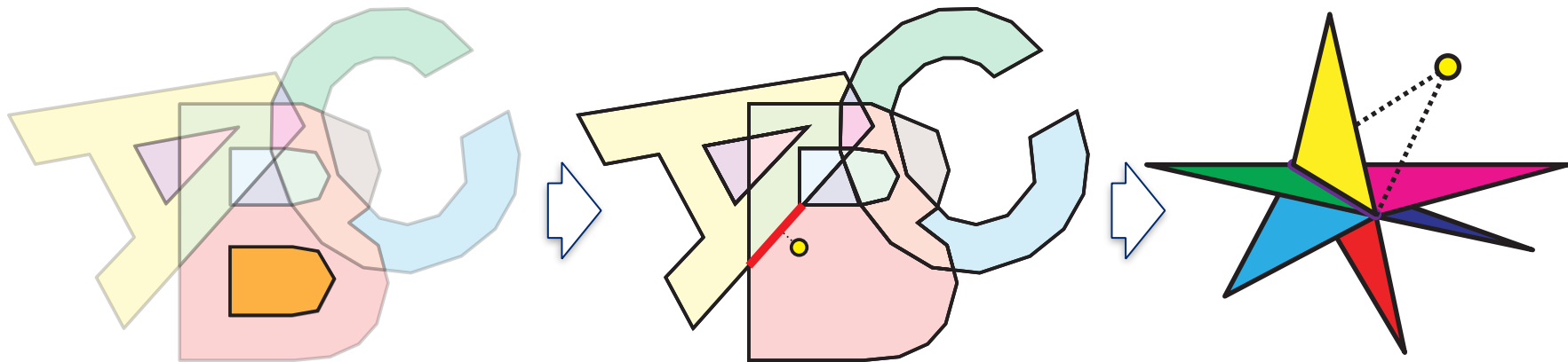
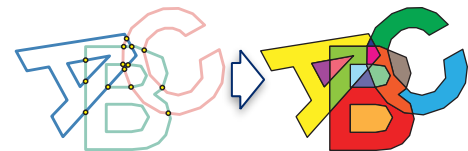
Extracting Cells



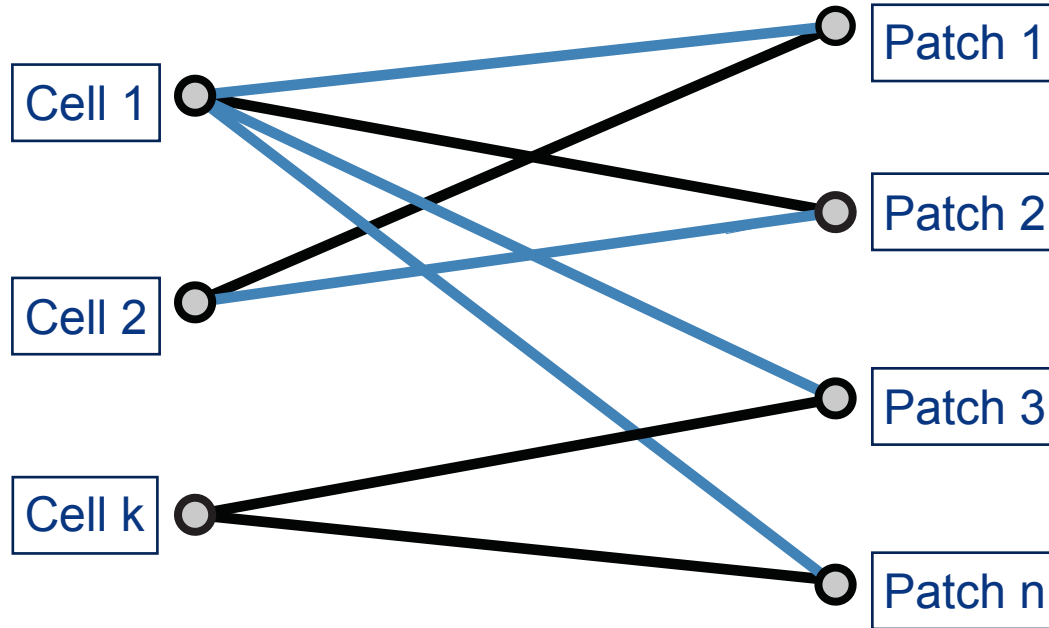
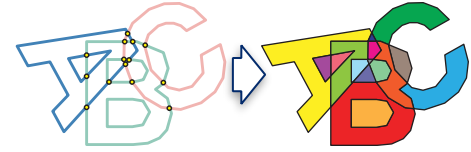
Extracting Cells



Extracting Cells



Extracting Cells

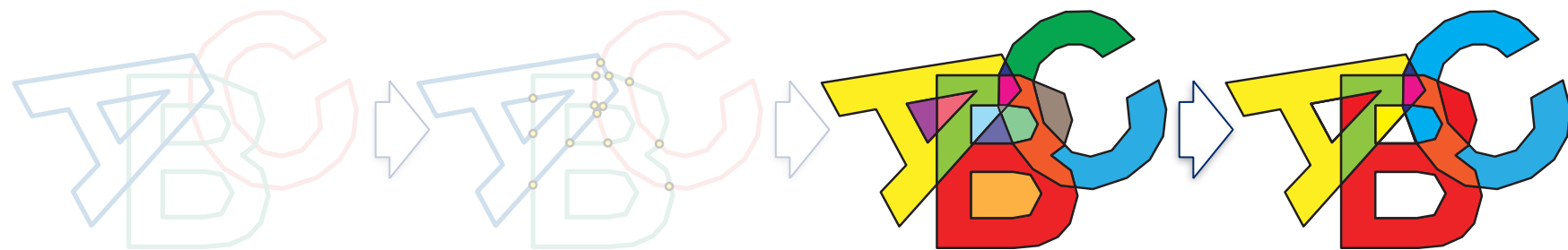


Algorithm

Resolving
(Self-)Intersections

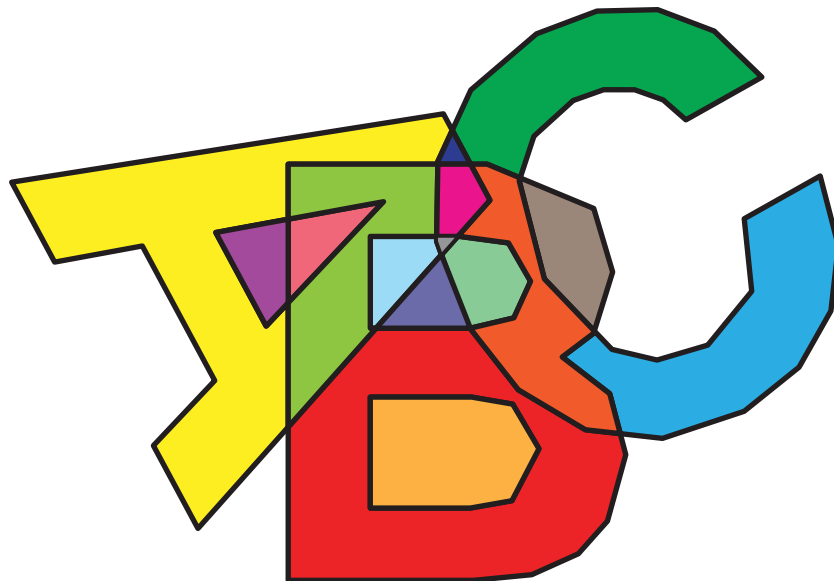
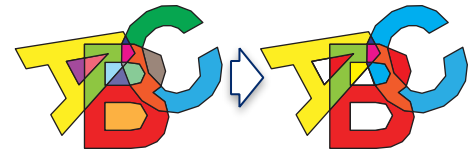
Extracting
Cells

Propagating
Winding Numbers

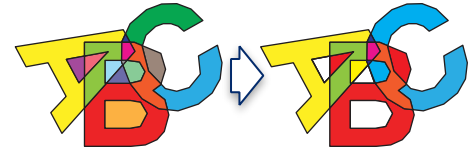


(0, 0, 0)
(1, 0, 0)
(0, 1, 0)
(0, 0, 1)
(1, 1, 0)
(0, 1, 1)
(1, 0, 1)
(1, 1, 1)

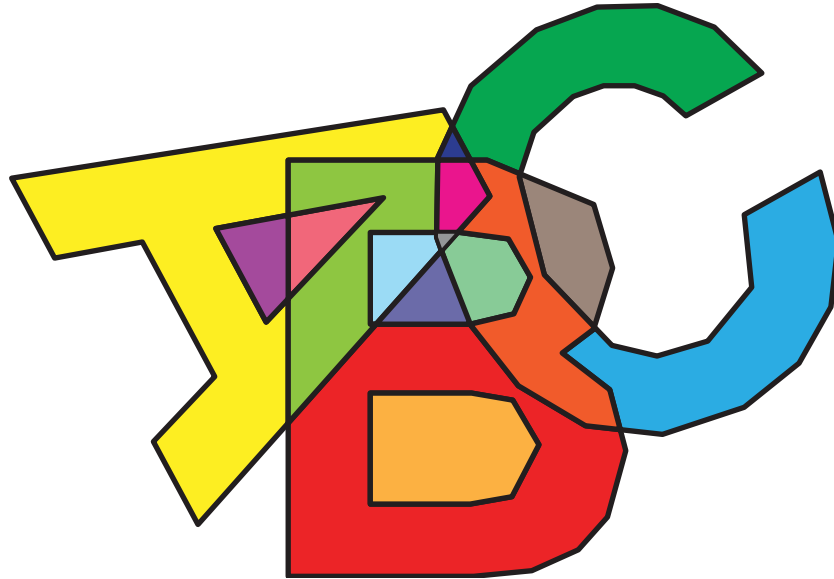
Propagating Winding Numbers



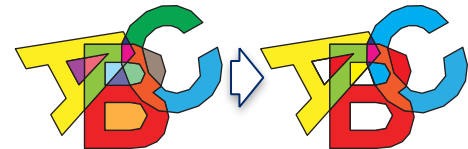
Propagating Winding Numbers



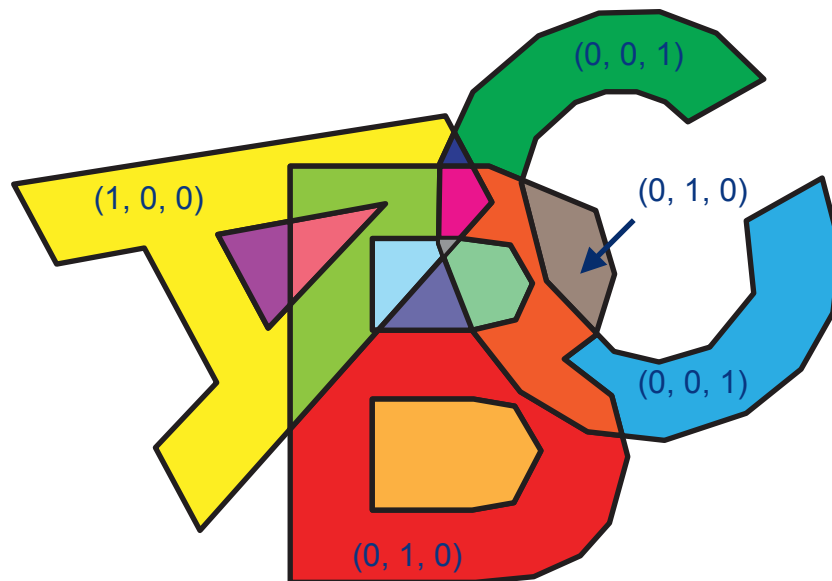
(0, 0, 0)



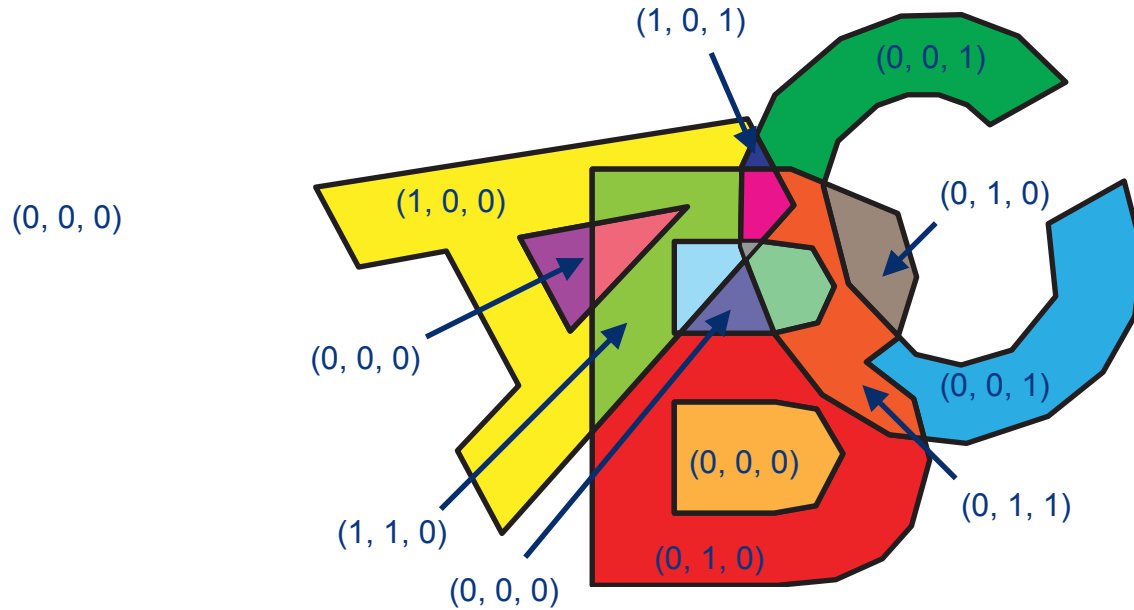
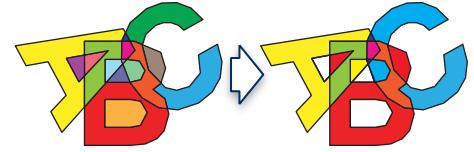
Propagating Winding Numbers



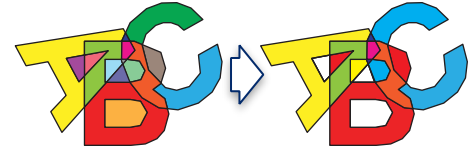
$(0, 0, 0)$



Propagating Winding Numbers

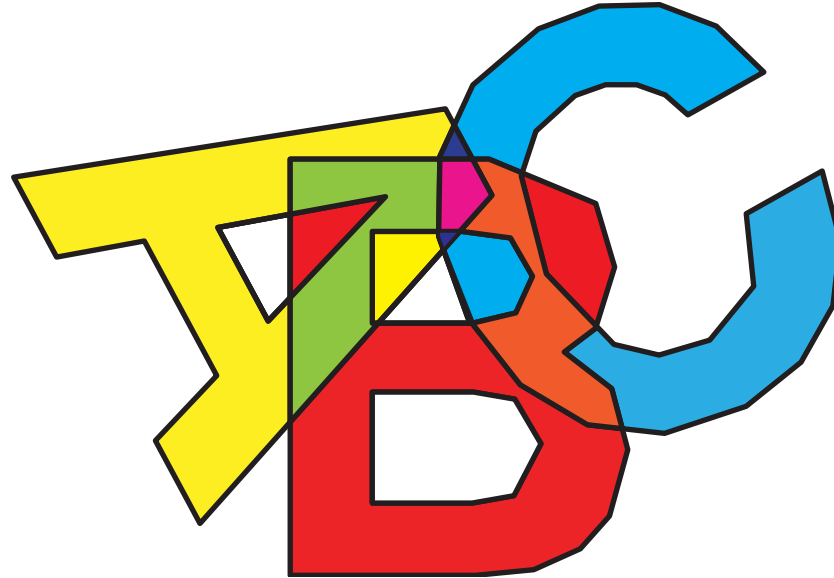


Propagating Winding Numbers



(w_A, w_B, w_C)

(0, 0, 0)
(1, 0, 0)
(0, 1, 0)
(0, 0, 1)
(1, 1, 0)
(0, 1, 1)
(1, 0, 1)
(1, 1, 1)



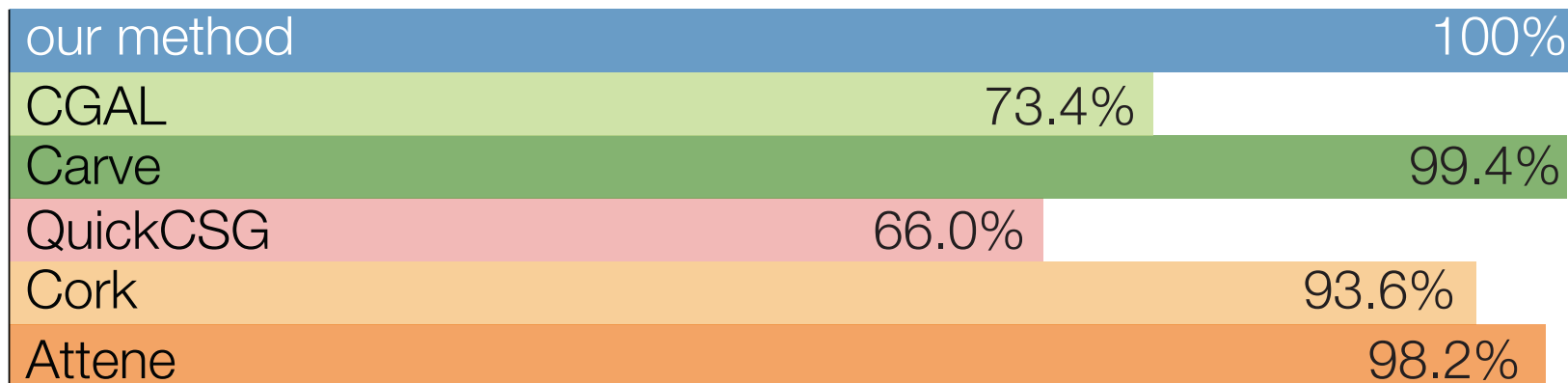
Results: Self Union

Check 1: produced an output

our method	100%
CGAL	73.4%
Carve	99.9%
QuickCSG	99.9%
Cork	94.1%
Attene	98.3%

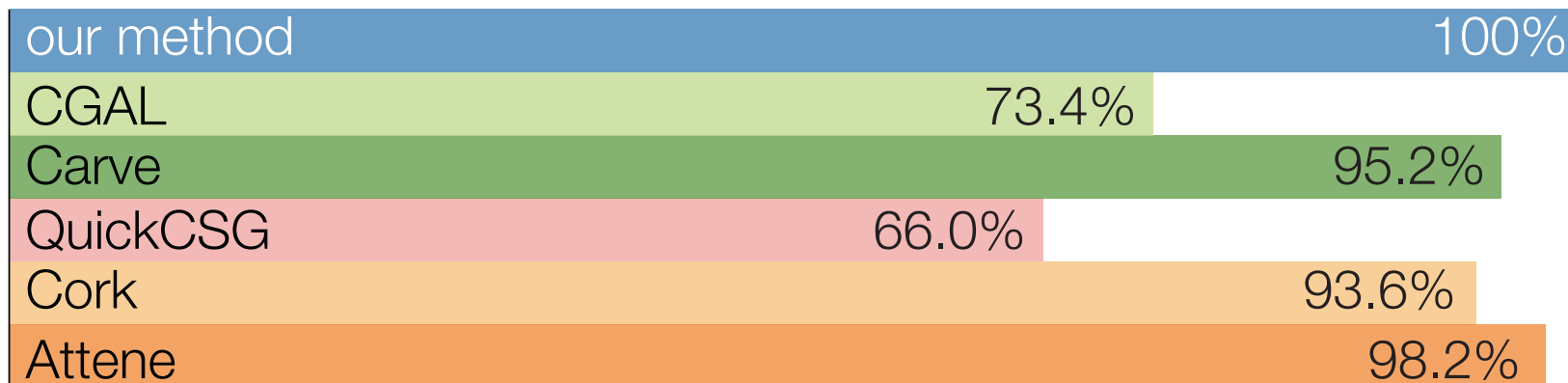
Results: Self Union

Check 2: output is closed



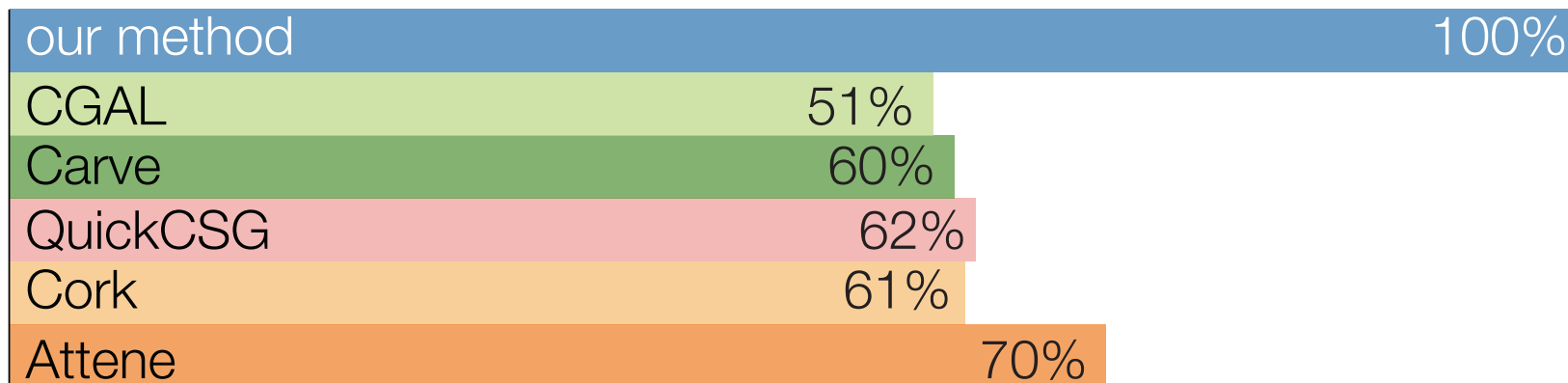
Results: Self Union

Check 3: output is PWN

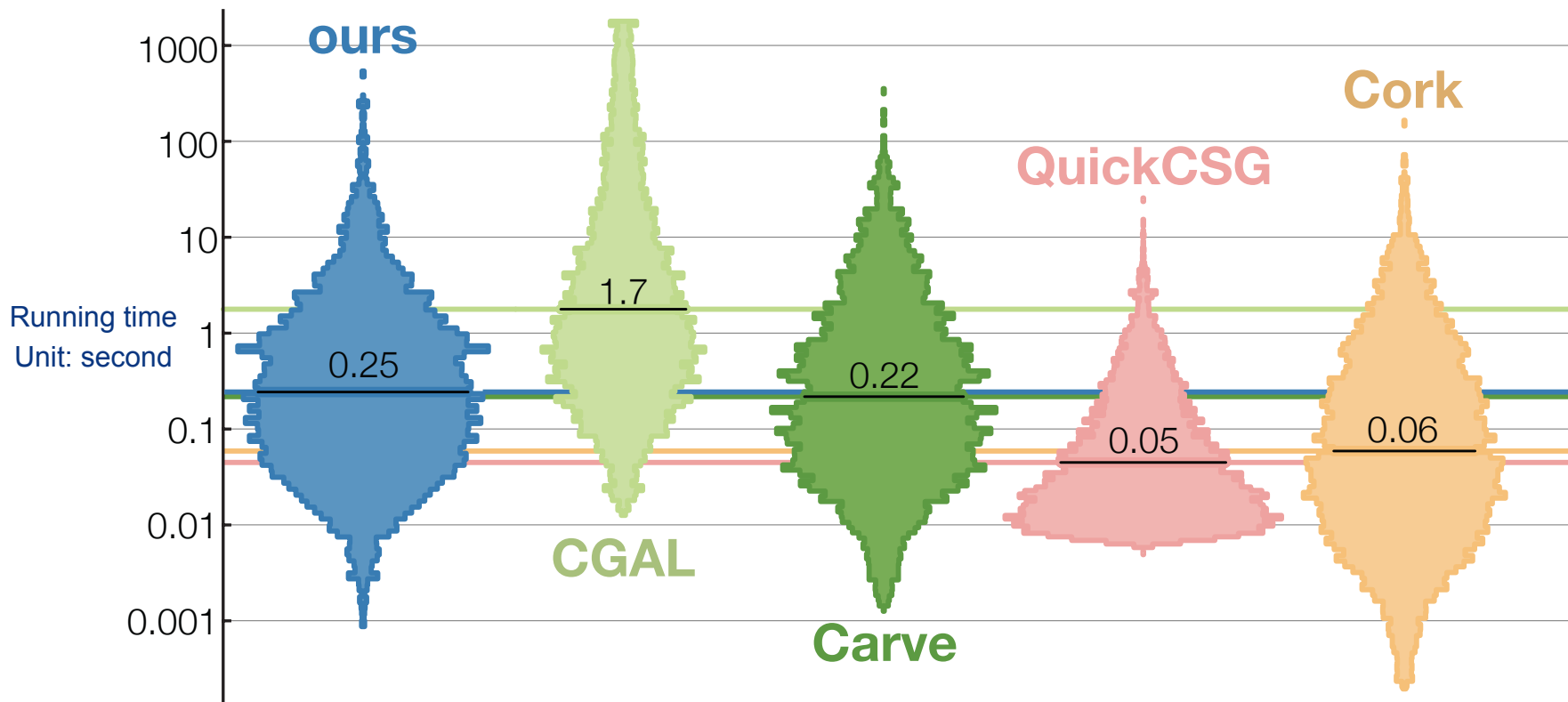


Results: Self Union

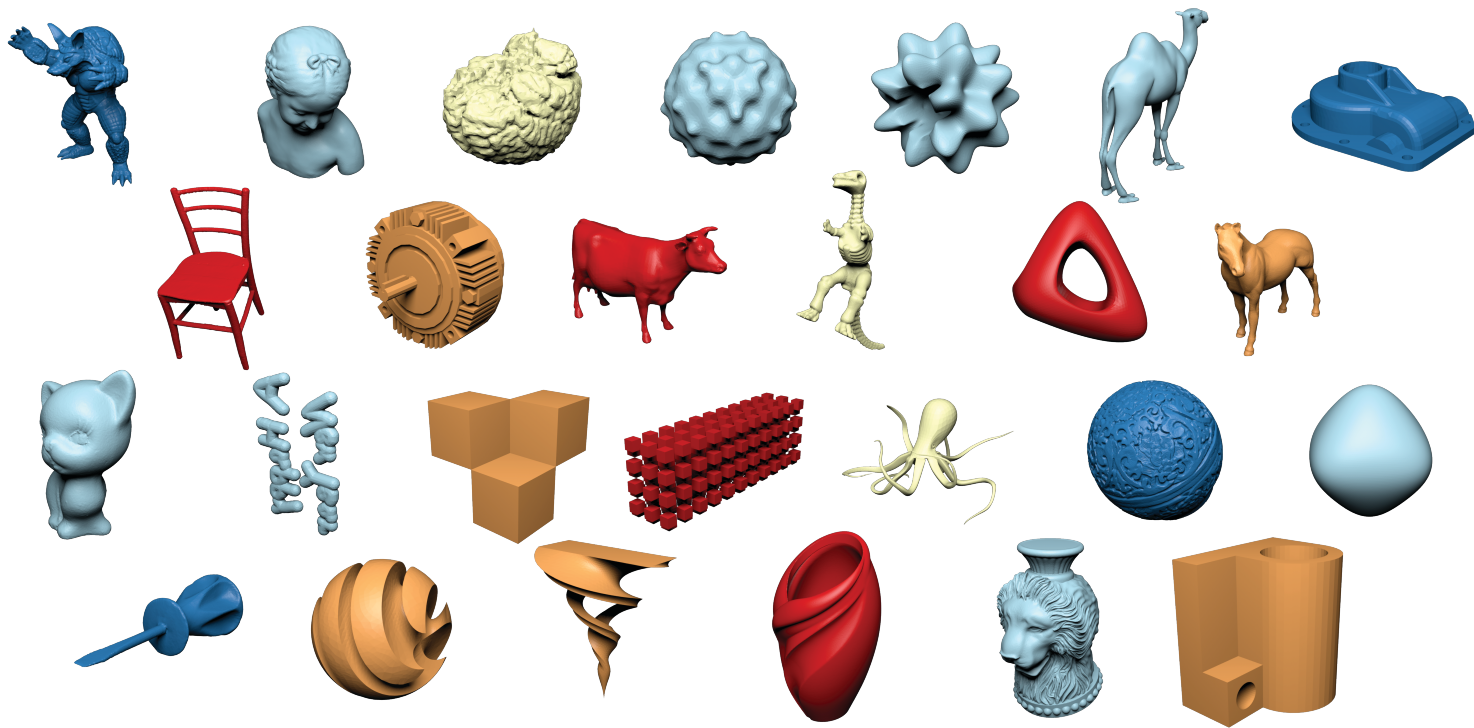
Check 4: output is self-intersection free



Results: Self Union



Results: [Barkai et al. 2015] Dataset

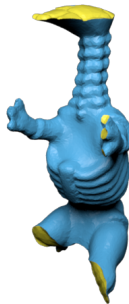


Results: [Barki et al. 2015] Dataset

$A \cup B$



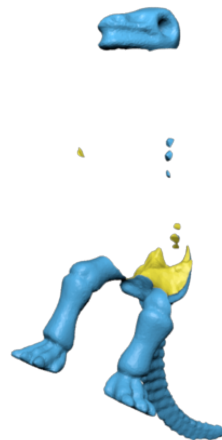
$A \cap B$



$A \setminus B$



$B \setminus A$



1404 results for all possible pairs.

Results: [Barki et al. 2015] Dataset



Results: [Barki et al. 2015] Dataset



Results: [Barki et al. 2015] Dataset



Results: [Barki et al. 2015] Dataset



Results: [Barki et al. 2015] Dataset



Results: [Barki et al. 2015] Dataset



Results: [Barki et al. 2015] Dataset



Results: [Barki et al. 2015] Dataset



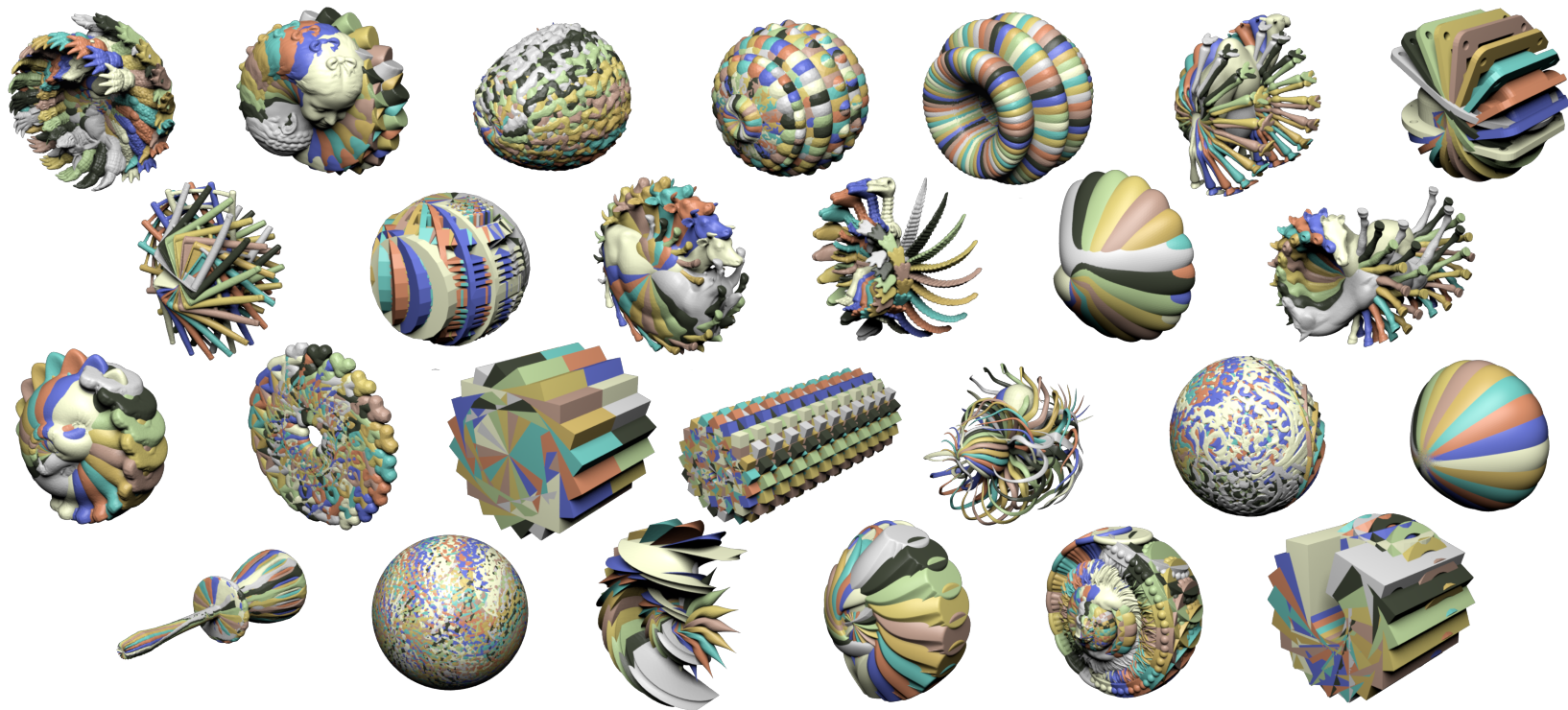
Results: [Barki et al. 2015] Dataset



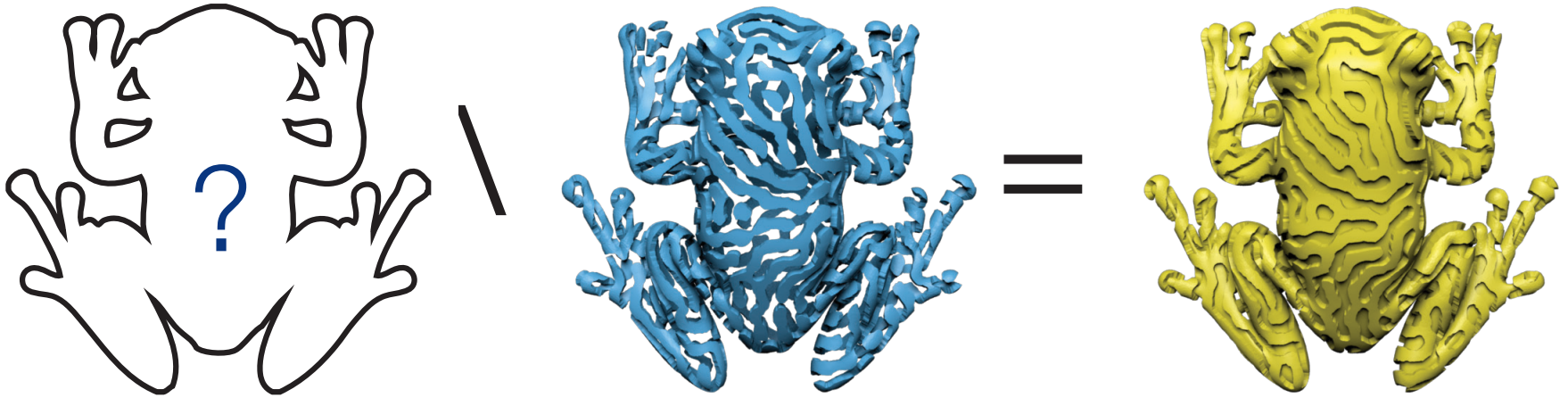
Results: [Barki et al. 2015] Dataset



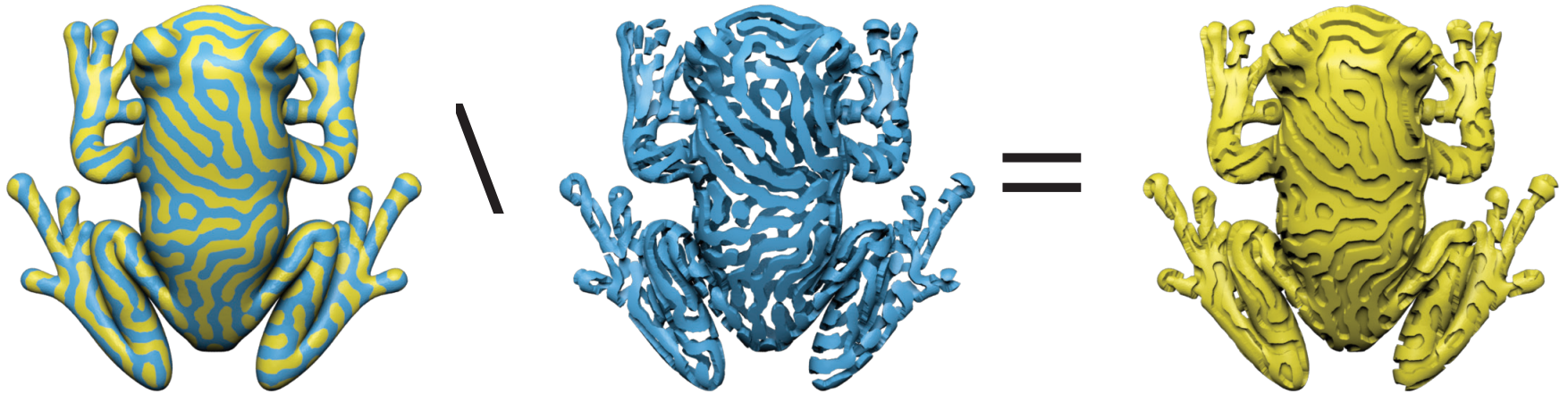
Results: [Barki et al.] Dataset



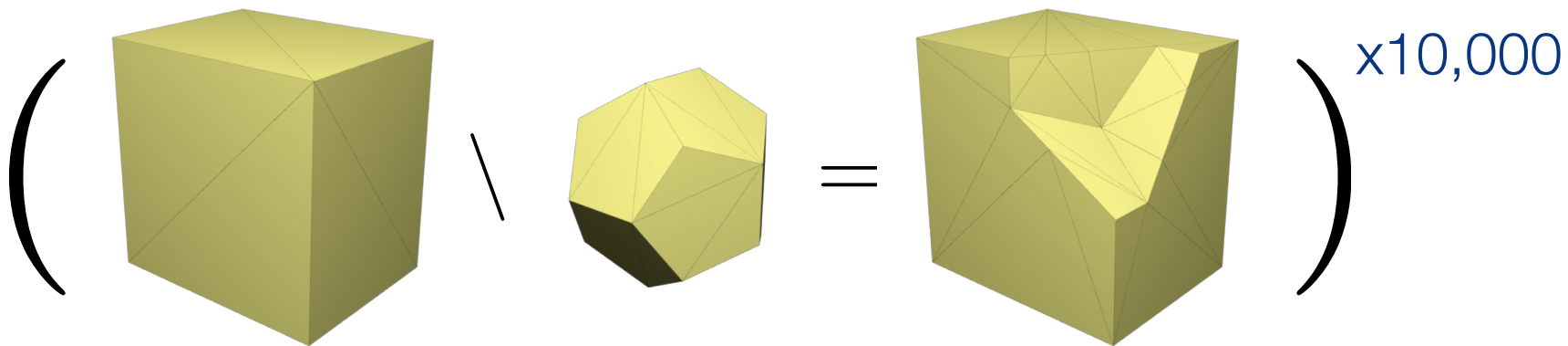
Result: Reverse Boolean



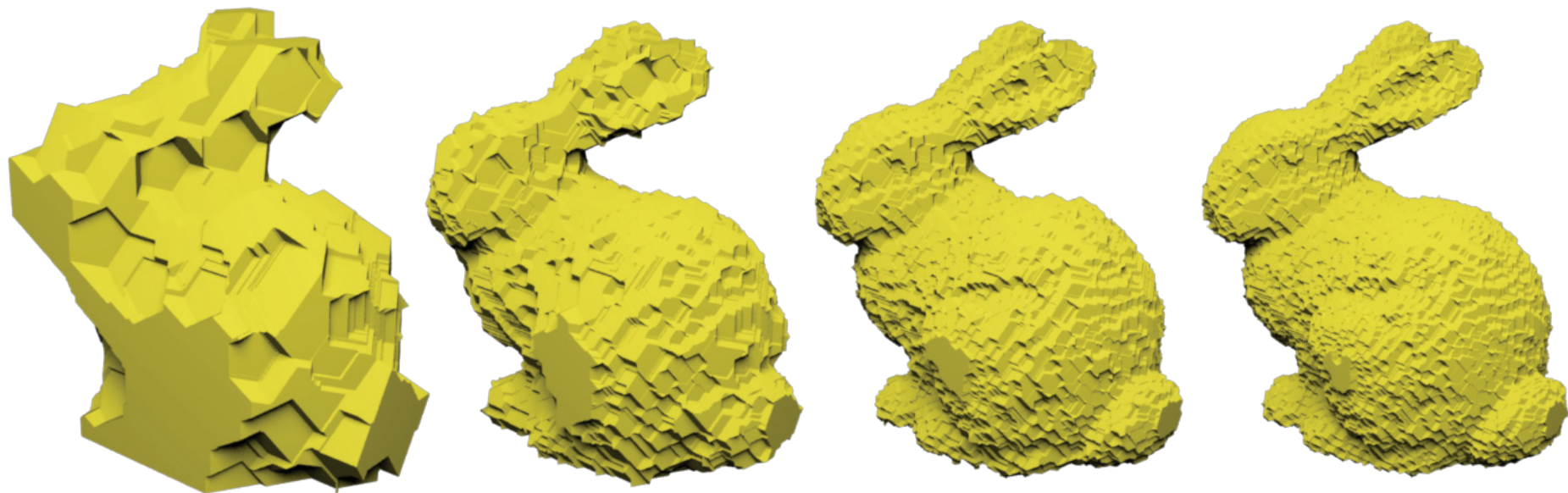
Result: Reverse Boolean



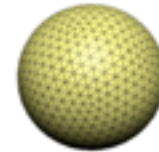
Results: Bunny Carve



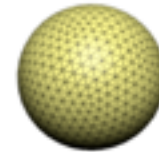
Results: Bunny Carve



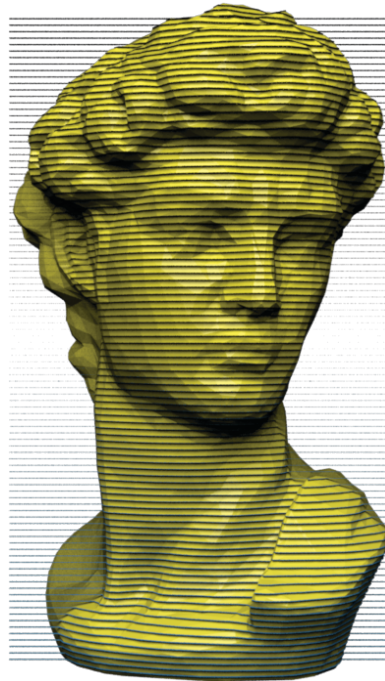
Results: Swept Volume



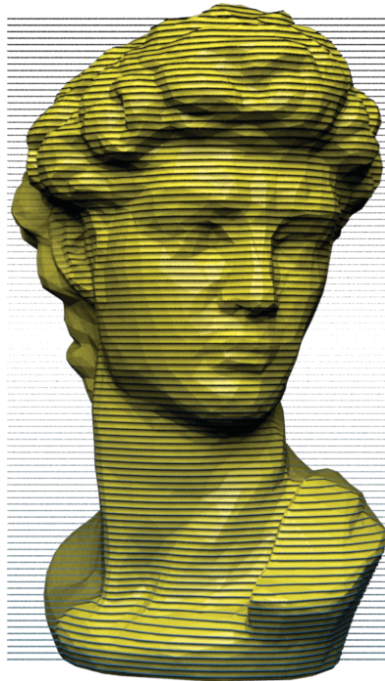
Results: Swept Volume



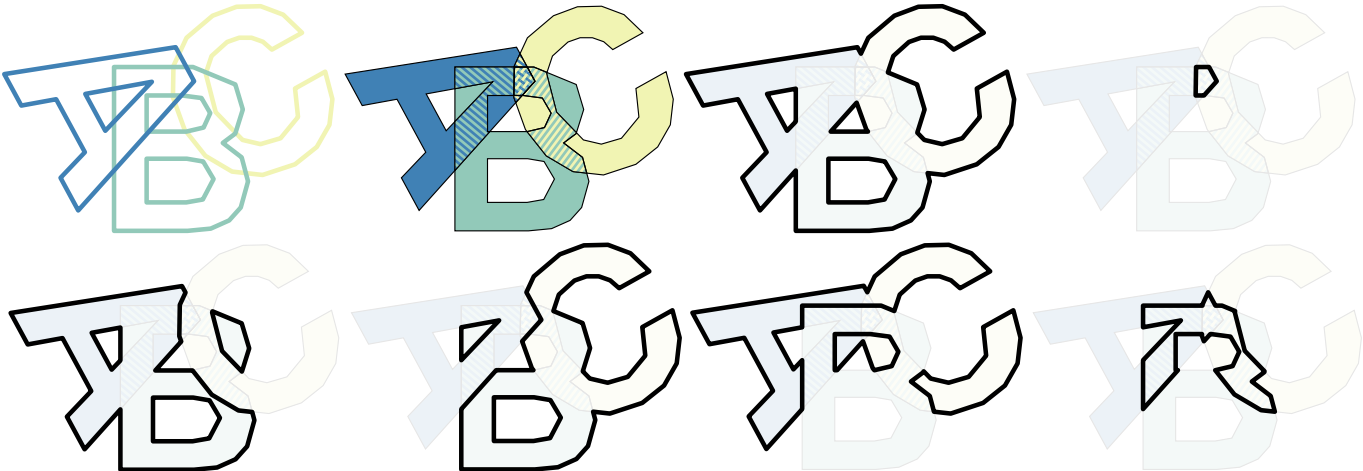
Results: CNC Milling



Results: CNC Milling

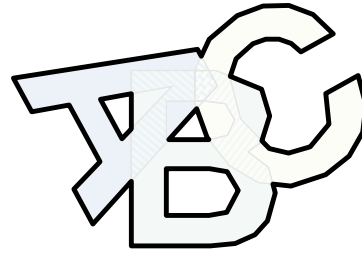
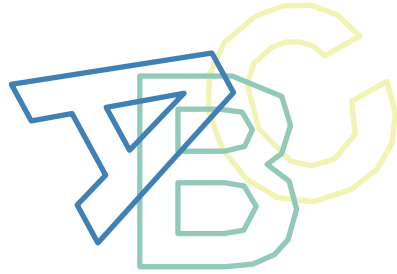


Conclusion

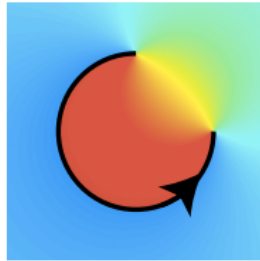


Future Works

- Boost performance with adaptive techniques.



- Repairing non-PWN meshes



Resources

Source Code

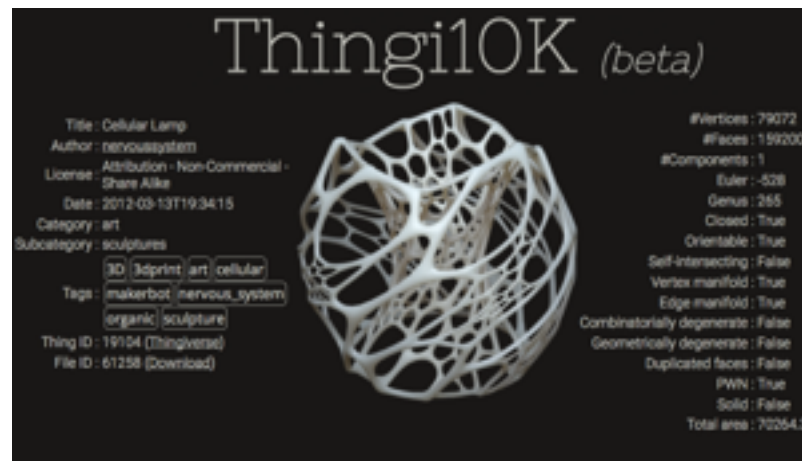
libigl

<https://github.com/libigl/libigl>

PyMesh

<https://github.com/qnzhou/PyMesh>

Dataset: Thingi10K



<https://ten-thousand-models.appspot.com/>