Robust Mesh Repair for 3D-Printing Models

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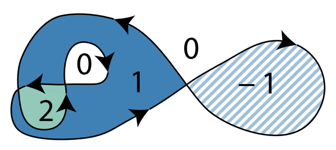
Winding Number

Winding number measures how many times a point is enclosed by a given surface. It is often used for determining if a point is inside of a 3D model. Mathmatically, 3D winding number is defined as:

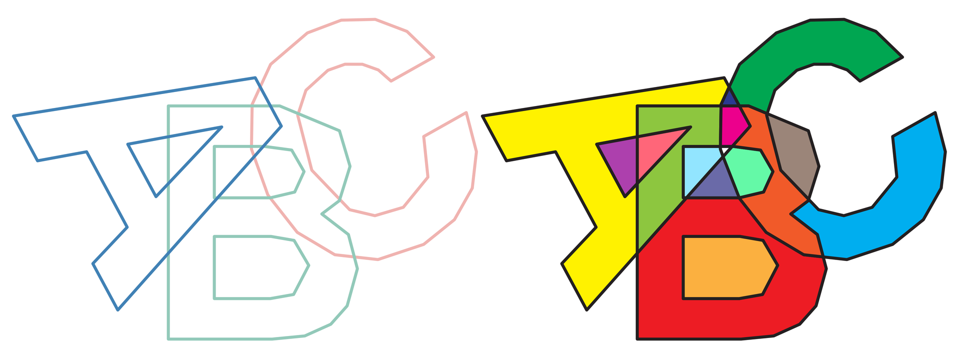


$$w(p) = \frac{1}{4\pi} \sum_{f \in F} \Omega_f(p)$$

where $\Omega_f(p)$ is the solid angle at point p in the tetrahedron formed by p and the triangle f. [Jacobson 2013] generalized winding number concept to open meshes, and demonstrated winding number is robust against common geometric artifacts.

Mesh Arrangements

A mesh arrangement is a collection of (possibly non-manifold, open boundary, self-intersecting, with degenerate faces, etc.) meshes partitioning the space into a number of cells.

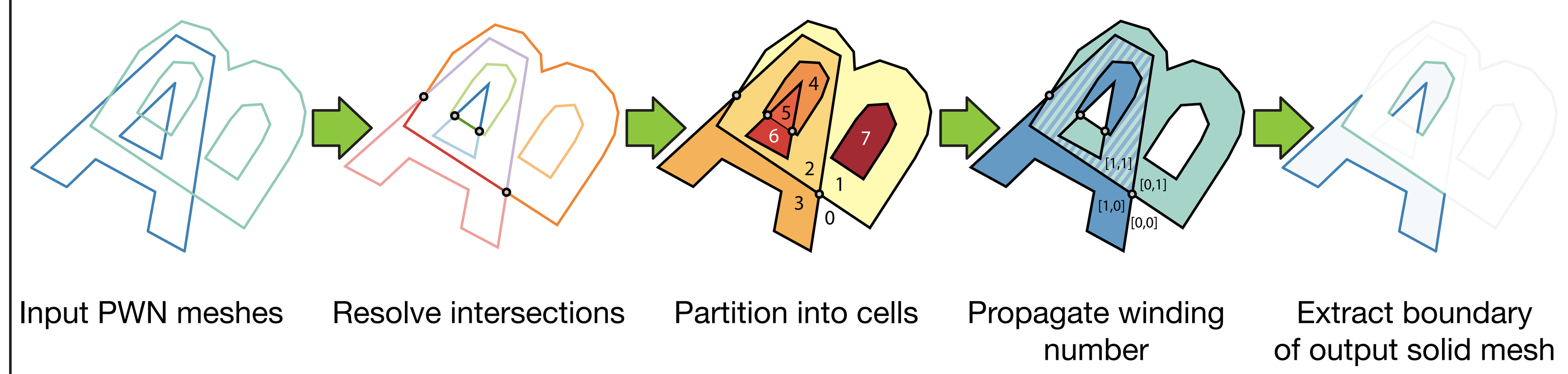


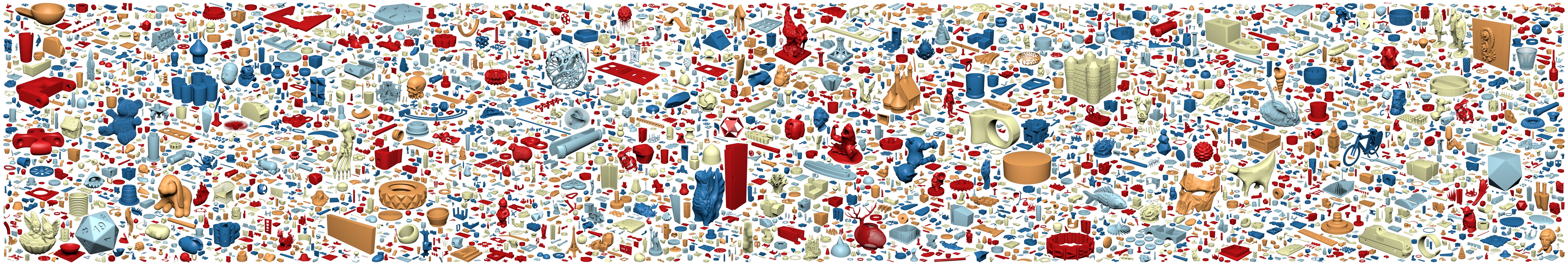
Key Concepts

PWN Mesh: is any mesh that induces a **P**iecewise-constant **W**inding **N**umber field. PWN mesh could contain multiple overlapping components with degenerate faces, self-intersections. PWN mesh can be non-manifold and topologically open. Our algorithm takes one or more PWN meshes as input.

Solid Mesh: is the non-degenrate boundary of a solid sub-region of R^3. It is a subset of PWN meshes that induces a {0,1} winding number field and is free of degenerate faces, self-intersections and duplicated faces. Out algorithm guarantees to output a solid mesh.

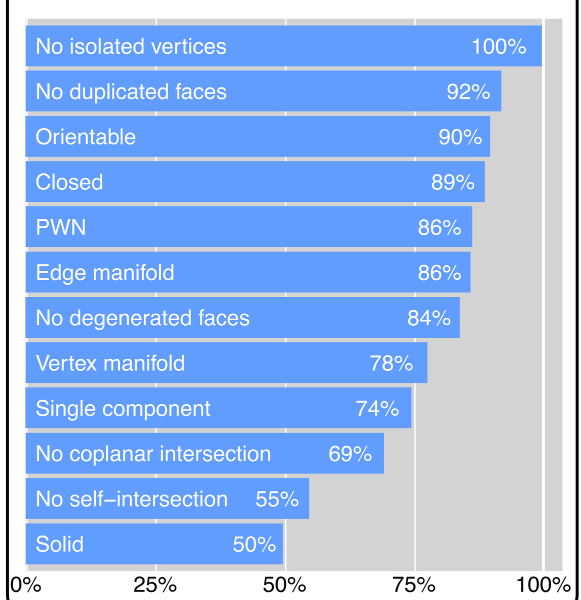
Pipeline





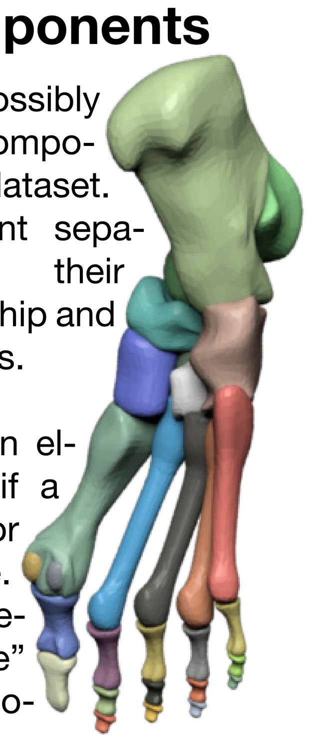
Dataset

We test our algorithm with 10,000 models from Thingiverse.



Multiple Components

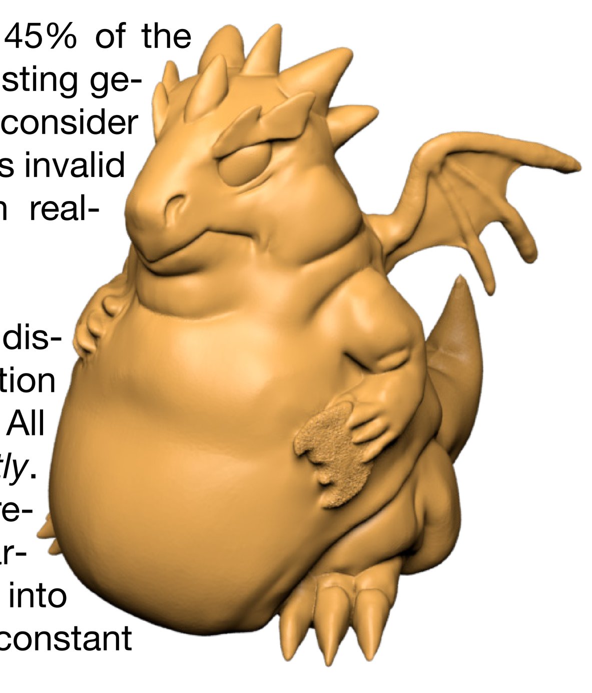
Models with multiple, possibly overlapping or nested, components make up 26% of our dataset. Processing each component separately disregards their nesting/overlapping relationship and may produce incorrect results.



Winding number provides an elegant way of determining if a given point is inside (|w|>0) or outside (|w|=0) of the shape. It is even possible to have regions that are "twice inside" due to overlapping components.

Self-Intersections

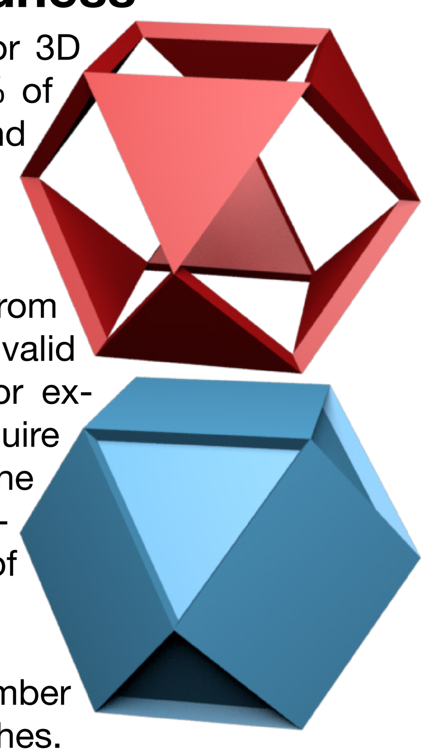
Self-intersection is detected in 45% of the models in our dataset. Most existing geometry processing algorithms consider meshes with self-intersections as invalid despite their large presence in real-world models.



In our algorithm, we make no distinction between self-intersection and mesh-mesh intersection. All intersections are resolved *exactly*. If the input mesh is PWN, the resolved mesh represents a valid arrangement that partition space into cells, each with piecewise-constant winding number.

Non-Manifoldness

Non-manifoldness is very common for 3D printing models. In our dataset, 14% of the models are not edge-manifold, and 22% of the models are not vertex-manifold.

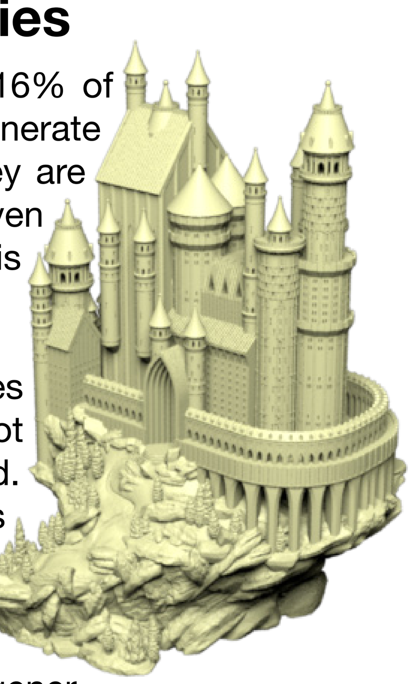


While non-manifoldness could arise from modeling errors, there are many valid usages of non-manifold meshes. For example, multi-material printers require separate mesh from each material. The example on the right is designed to illustrate triangles and squares of cuboctehedron in different material.

Mesh arrangements and winding number are well-defined for non-manifold meshes.

Degeneracies

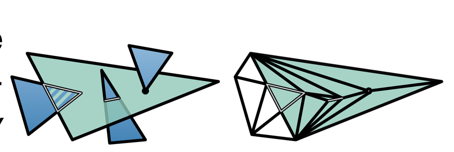
Exactly Degenerate faces appear in 16% of the models in our dataset. Nearly degenerate faces are even more ubiquitous. They are particularly hard to handle because even basics quantities such as face normal is not well-defined or numerically stable.



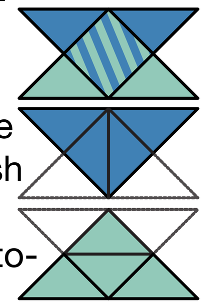
In our settings, exactly degenerate faces can be ignored because it does not effect the induced winding number field. The resulting topological seam is closed when resolving intersections. Because all intersections are computed exactly, and our algorithm does not rely on surface normal. Nearly degenerate faces does not pose any challenges.

Intersection Resolution

The intersection of two triangles can be empty, a point, a segment or a convex polygon. Intersection must be compute *exactly* to avoid trouble down the pipeline. Specifically, rounding to float may re-introduce intersections, create degeneracies or rotate an edge by upto 90 degrees. General positioning assumption is not valid for 3D printing models, where 31% of the input contain coplanar intersections.

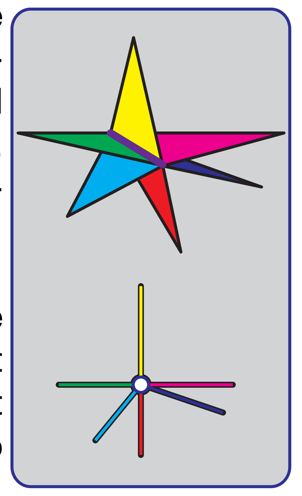


Once intersection is computed, triangles involved must be splitted so that intersection are represented by mesh vertices/edges/faces. Because constrained delaunay triangulation is not unique, coplanar faces must be triangulated together.



Facet order around an edge

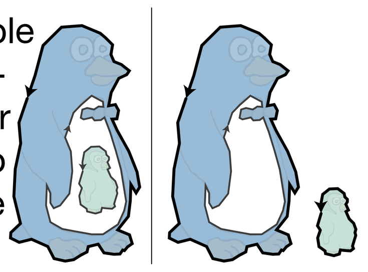
To correctly construct mesh arrangement, one must extract all faces boudning a given partition or R^3, which relies on consistent cyclical order of facets around all non-manifold edges. Computing such ordering is misleadingly innocuous.



Due to the presence of nearly degenerate faces, surface normals are not reliable. Facet ordering must be computed solely with exact predicates. Symbolic perturbation is used to break ties caused by duplicated faces.

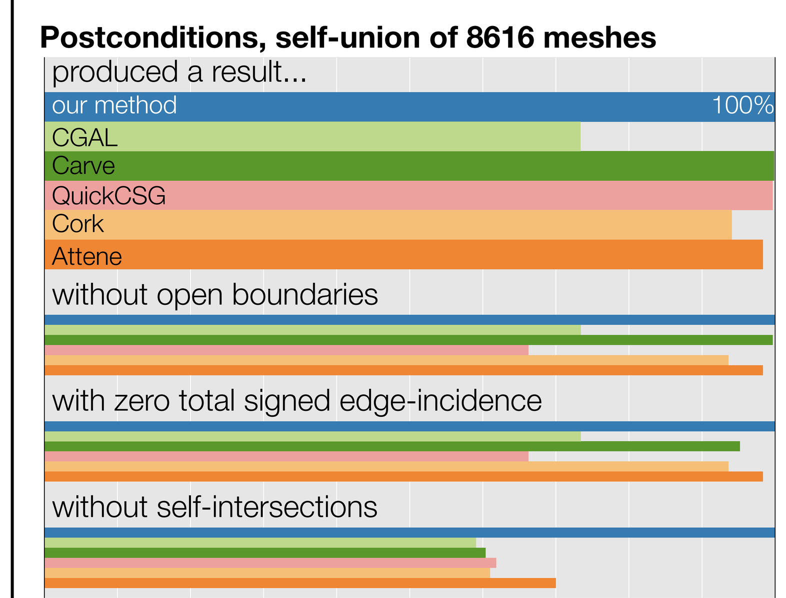
Nesting Relationship

For mesh with multiple components, it is important to determine their nesting relationship so winding number can be correctly propagated.



For each pair of components (c1, c2), we find the closest object on c2 to a point, p, sampled from c1. We show that all cases can be reduced to determine the cyclical order of a pivot facet created by connecting p with an edge that touches the closes object.

Results



Resources

Code:
Mesh boolean: https://github.com/libigl/libigl
Comparison: https://github.com/qnzhou/PyMesh

Dataset:
https://ten-thousand-models.appspot.com/

Publication:

Zhou, Q., Grinspun, E., Zorin, D., Jacobson, A. 2016. Mesh arrangements for solid geometry. Conditionally accepted to ACM SIGGRAPH.

Zhou, Q., Jacobson, A. 2016. Thingi10K: A dataset of 10,000 3D-printing models. Submitted to SGP.