Short Papers

Telecentric Optics for Focus Analysis

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Abstract—Magnification variations due to changes in focus setting pose problems for vision techniques, such as, depth from focus and defocus. The magnification of a conventional lens can be made invariant to defocus by simply adding an aperture at an analytically derived location. The resulting optical configuration is called "telecentric." It is shown that most commercially available lenses can be turned into telecentric ones. The procedure for calculating the position of the additional aperture and a detailed analysis of the photometric and geometric properties of telecentric lenses are presented. Experiments are reported that use a phase-based shift detection algorithm to demonstrate the magnification invariance of telecentric lenses.

Index Terms—Commercial lenses, telecentric optics, constant magnification imaging, aperture placement, phase-based motion estimation, depth from focus/defocus.

1 Introduction

The problem of magnification variation due to change in focus setting has significance in machine vision. The classical approach to solving this problem has been to view it as one of *camera calibration* [9]. Willson and Shafer [9] conducted a careful analysis of the interaction between focus and magnification. They proposed a joint calibration approach that measures the relation between zooming and focusing for a given lens. Using this calibration model, it becomes possible to hold magnification constant while focusing; the calibration results are used to apply zoom adjustments so as to correct magnification changes caused by focusing. Though this approach provides a general scheme to tackle the problem, it has its drawbacks. One requires an expensive computer-controlled zoom lens and the extensive calibration procedure mentioned above, even if one only needs to vary the focus setting and not any other parameter. Further, the necessity to change two physical parameters (focus and zoom) simultaneously tends to increase errors caused by backlashes in the lens mechanism and variations in lens distortion.

An alternative approach to the magnification problem is a computational one, commonly referred to as image warping. Darrell and Wohn [2] proposed the use of warping to correct image shifts due to magnification changes caused by focusing. This method is simple and effective for some applications, but can prove computationally intensive for real-time ones. Furthermore, since warping is based on spatial interpolation and resampling techniques, it could introduce undesirable effects such as smoothing and aliasing. These can be harmful for applications that rely on precise spatial-frequency analysis, such as depth from focus / defocus.

Depth from focus / defocus methods provide a powerful means of getting a range map of a scene from two or more images taken from the same viewpoint but with different optical settings. For instance, depth from focus uses a sequence of images taken by incrementing the focus setting in small steps. For each pixel, the focus setting that maximizes image contrast is determined. This in turn can be used to compute the depth of the corresponding scene point. Magnification variations due to defocus, however, cause additional image variations in the form of translations and scalings. Estimation of image contrast in the presence of these effects will clearly result in depth errors. This problem proves even more detrimental in the case of depth from defocus, where the number of images used for depth estimation can be as few as two.

In this paper, a simple but effective approach to constant-magnification imaging is described. The magnification problem is eliminated in its entirety by the use of an optical configuration, referred to as *telecentric optics*. Though telecentricity has been known for long in optics [1], [4], it has not been exploited in the realm of computational vision. There are a few commercially available telecentric lenses [6] but these are telecentric on the object side and not the image side of the lens. Telecentricity on the object side implies that the image projection model is orthographic. This is different from telecentricity on the image side where the image projection model remains perspective (and hence wide in field of view) but yet the magnification of scene points is constant with respective to the location of image detector behind the lens. It is image-side telecentricity that we seek in this paper.

In image-side telecentricity, magnification remains constant despite focus changes. We show how commercially available lens (used extensively in machine vision) are easily transformed to telecentric ones by adding an extra aperture. We analytically derive the positions of aperture placement for a variety of off-the-shelf lenses. Further, extensive experimentation is conducted to verify the invariance of magnification to defocus in four telecentric lenses that were constructed from commonly used commercial lenses. These experiments make use of a phase-based optical flow algorithm that measures local image shifts between frames with subpixel accuracy. We have successfully incorporated a telecentric lens into a real-time active range sensor that is based on depth from defocus [5]. The application of telecentric optics to passive depth from defocus is demonstrated in [8] and [7].

2 Telecentricity

2.1 Conventional Lens Model

To begin with, we discuss the lens model that is widely used in computer vision. Fig. 1 shows the commonly used image formation model, where the main assumptions are that the lens is thin and the aperture position coincides with the lens. All light rays that are radiated by scene point P and pass the aperture A are refracted by the lens to converge at point Q on the image plane. The relationship between the object distance d, focal length of the lens f, and the image distance d_i is given by the Gaussian lens law: $\frac{1}{d} + \frac{1}{d_i} = \frac{1}{f}$.

Each point P on the object plane is projected onto a single point Q on the image plane, causing a clear or *focused* image I_f to be formed. When the focus setting is changed by displacing the sensor plane, for example, to I_1 or I_2 from I_f, the energy received from P by the lens is distributed over a circular patch on the sensor plane.[1] Although this causes a blurred image, the effective image

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1. Here, focus change is modeled as a translation of the sensor plane. This model is also valid for the case of lens translation which is used in most non-zoom lens systems, where the distance d between lens and object is typically much larger than the focal length f.

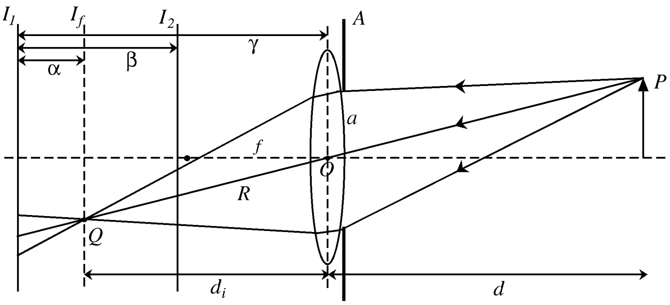


Fig. 1. Image formation using a conventional thin lens model.

location of point P can be taken to be the center of the circle. This center lies on the ray R which is radiated from point P and passes through the center of the aperture O and refracted by the lens. This ray is called the *principal ray* [1], [4]. Since its intersection with the sensor plane varies with the position of the sensor plane, image magnification varies with defocus.

2.2 Telecentric Optics

Keeping in mind the image formation model shown in Fig. 1, we proceed to discuss the constant-magnification configuration, called *telecentric optics*. Fig. 2 illustrates the principle underlying telecentric projection. The only modification made with respect to the conventional lens model of Fig. 1 is the use of the external aperture A'. The aperture is placed at the *front-focal plane*, i.e., a focal length in front of the *principal point O* of the lens. This simple addition solves the problem of magnification variation with distance α of the sensor plane from the lens. Straightforward geometrical analysis reveals that the ray of light R' from any scene point that passes through the center O' of aperture A', i.e., the *principal ray*, emerges parallel to the optical axis on the image side of the lens [4]. Furthermore, this parallel ray is the axis of a cone that includes all light rays radiated by the scene point, passed through by A', and intercepted by the lens. As a result, despite defocus blurring, the effective image coordinates of point P on the sensor plane stay constant irrespective of the displacement of the sensor plane from I_f. In the case of depth from defocus, the magnification of any point in the scene, regardless of its position in the scene, remains the same in both images, I_1 and I_2. It is also easy to see from Fig. 2 that this constant-magnification property is unaffected by the aperture radius a', as far as it is not large enough to cause severe vignetting.

2.3 Properties of Telecentric Optics

While the nominal and effective *F-numbers* for the conventional lens model in Fig. 1 are $f / 2a$ and $d_i / 2a$, respectively, they are both equal to $f / 2a'$ in the telecentric case. The reason for this invariance of the F-number for telecentric optics is the following: The effective F-number is defined as the ratio of the height to the diameter of the base of the light cone that emerges out of the lens.

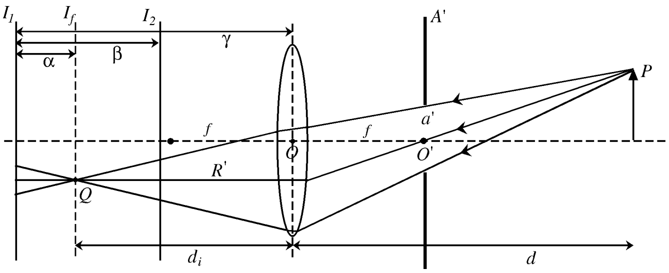


Fig. 2. Telecentric optics achieved by adding an aperture to a conventional lens. This simple modification causes image magnification to be invariant to the position of the sensor plane, i.e., the focus setting.

The nominal F-number is the effective F-number when the lens is focused at infinity. The above-mentioned light cone is bordered by the family of *marginal rays* which pass through the aperture A' touching its circumference. Consider these rays as they emerge from the lens on the image side. If the scene point P is displaced, the marginal rays on the image side only shift in a parallel fashion, keeping the angle subtended by the apex of the cone constant. This is due to the aperture being located at the front focal plane (see Fig. 2): Light rays which pass through the same point in the front focal plane emerge parallel from the lens. Since the light cone formed behind the lens remains the same and only its apex shifts with the location of the scene point, the effective F-number is constant.

The above fact results in another remarkable property of the telecentric lens: The image brightness stays constant in a telecentric lens, while, in a conventional lens, brightness decreases as the effective focal length d_i increases. This becomes clear by examining the image irradiance equation, see [3]:

$$E = L \frac{\pi}{4} \frac{\cos^4 \theta}{F_e^2} \tag{1}$$

where E is the irradiance of the sensor plane, L is the radiance of the surface in the direction of the lens, and F_e is the effective F-number. In the case of a conventional lens, $F_e = d_i / 2a$, while $F_e = f / 2a'$ for a telecentric lens. θ is the angle between the optical axis and the principal ray which originates from object point P and passes through the center of the aperture.

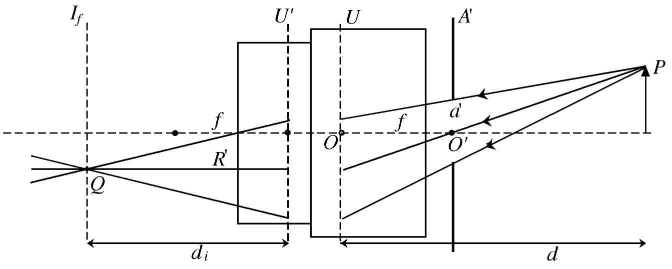
In summary, telecentric optics provides us a way of taking multiple images of a scene at different focus settings while keeping magnification constant between the images. In practice, this can be accomplished by using a beam splitter (or for more than two images, a sequence of beam splitters) behind the telecentric lens. In a real-time application, all images can be digitized simultaneously and processed in parallel as in [5].

3 Aperture Placement

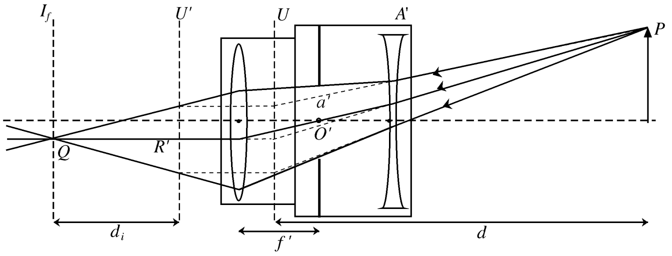
Although the discussion on telecentricity in Section 2 was based on the thin lens model, the results holds true for compound lenses. The thin lens is just a special case when the two *principal planes* [1], [3] coincides. Fig. 1 is easily modified for the compound lens case, by replacing the thin lens with the two principal planes, U and U', of the compound lens, as shown in Fig. 3a. The Gaussian lens law remains valid when the distances are measured as follows: the scene point distance d from plane U and the image distance d_i from plane U'.

Given an off-the-shelf lens, the additional telecentric aperture can be easily appended to the casing of the lens as far as the front focal plane is outside the lens. This is the case with most lenses including telephoto ones. However, for wide angle lenses with focal lengths shorter than the *back focal length* (distance from lens mount plane to sensor plane), the front focal plane is likely to be inside the lens. In such cases, one can still make the lens telecentric by placing an aperture inside the lens. The procedure is as follows. First, consider a set of parallel rays entering from the image side of the lens and find the point in the lens system where the parallel rays converge. In Fig. 3b, this would correspond to the point O'. If this point does not lie inside any of the physical lenses that comprise the compound lens, one can open the lens and place the telecentric aperture at the plane passing through the point and normal to the optical axis of the lens. Fujinon's CF12.5A f/1.4 is an example such a lens, which we converted to telecentric by placing an aperture inside and used to develop a real-time focus range sensor [5].

In practice, the exact location where the additional aperture needs to be placed can be determined in the following way. In some cases, the lens manufacturer provides information regarding the front focal position, which is customarily denoted as F in the schematic diagram. For instance, Nikon provides this data for their old line of SLR lenses, two of which, Micro-Nikkor 55 mm f/2.8 and Nikkor 85 mm f/2, are used in our experiments reported in Section 5.1. If this information is not available from the manufacturer, it can be determined by the following procedure. Hold the lens between a screen (say, a white piece of paper) and a far and bright source (such as the sun or a distant lamp). In this setup the lens is held in the direction opposite to normal use; light enters from the back of the lens and the image is formed on the screen in front of the lens. The screen is shifted around to find the position that provides the clearest image. This position is the front focal plane where the telecentric aperture should be placed.



(a)



(b)

Fig. 3. Telecentric aperture placement in the case of compound lenses. (a) Aperture placement outside the lens. (b) Aperture placement inside the lens.

The lens is then mounted on an image sensor with the telecentric aperture attached to it, and the actual magnification variation due to defocus is used as feedback to refine the aperture position so as to drive magnification variation to zero. The method we used to measure magnification change is detailed in Section 5.1. The above refinement process is recommended even when the front focal plane position is available from the manufacturer. This is because the precise position of the front focal plane may vary slightly between lenses of the same model. As we will see, after conversion to telecentric, magnification variations produced by a lens can be reduced to well below 0.1 percent.

While using the telecentric lens, the original aperture of the lens should be opened fully and the diameter of the telecentric aperture should be chosen so as to minimize vignetting. The degree of vignetting can be gauged by reducing the original aperture by one step from the fully open position and measuring image brightness in the corners of the image. If vignetting is significant, even this small reduction in the original aperture size will change image brightness. Table 1 summarizes all the information needed to convert five popular lenses into telecentric ones. We have converted all five lenses and the telecentric versions of three of them are shown in Fig. 4. In each case, the telecentric aperture resides in the aluminum casing attached to the lens. In the case of Fujinon's CF12.5A, the telecentric aperture resides inside the lens body.

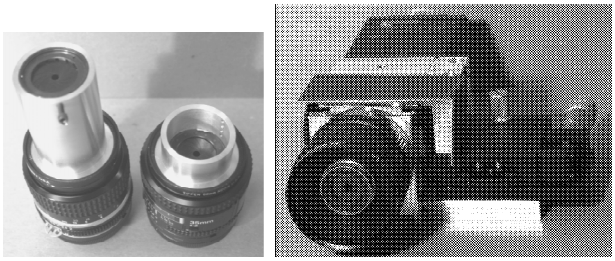
TABLE 1
APERTURE PLACEMENT FOR FIVE OFF-THE-SHELF LENSES

Lens	Focal length	F-number	Aperture position	Min. F-number†
Fujinon CF12.5A	12.5 mm	1.4	Inside : 4 mm	8
Cosmicar B1214D-2	25 mm	1.4	Outside: 4.7 mm	8
Nikon AF Nikkor	35 mm	2	Outside: 3.3 mm	8.5
Nikon Micro-Nikkor	55 mm	2.8	Outside: 46 mm (38.5 mm)*	13
Nikon Nikkor	85 mm	2	Outside: 67 mm (57.8 mm)*	6.8

Note. Outside aperture position is measured along the optical axis from the surface of outermost lens. Inside aperture position is measured from the stray light stop aperture toward the scene direction. In each case, the minimum F-number (right column) corresponds to the maximum aperture for which there is no vignetting.

** Number in () is the maker supplied front focal position.*

† Min. F-number corresponds to the maximum aperture that does not cause vignetting.



(a) (b)

Fig. 4. Popular lenses converted into telecentric ones. (a) The Nikon Nikkor f = 85 mm SLR lens (left) and the Nikon AF Nikkor f = 35 mm SLR lens (right). (b) Fujinon CF12.5A f = 25 mm lens on a micrometer stage. In each case, the telecentric aperture resides in an aluminum casing attached to the lens. The micrometer stage in (b) allows constant-magnification focus variation by displacing the image sensor from the lens.

4 OTHER FORMS OF TELECENTRICITY

There is another way to convert a conventional lens into a telecentric one. This is by placing a convex lens between the lens and the image sensor to make the principal ray parallel to the optical axis. Though this is better in the sense that one can use the full aperture range without worrying about vignetting, it changes the focal length and the position of the image plane. In addition, the convex lens must reside deep inside the camera.

It is worth mentioning that there are some commercially available lenses that are called telecentric (see [6], for example). But these lenses are telecentric on the object side, i.e., principal rays come into the lens parallel to the optical axis, which is opposite to what has been discussed here. These lens are used in profile projectors where magnification changes caused by the variation of object distance from the lens is a serious problem. In effect, these lenses realize precise orthographic projection and not constant-magnification focusing.

It turns out that zoom lenses for 3-CCD color cameras are made telecentric on the image side to avoid color shading caused by RGB color separation. One can tell this by looking at the exit pupil position in the specification sheet provided by the manufacturer. If the exit pupil position is at ∞, the lens is image-side telecentric [1], [4]. An example is Fujinon's H12 $\times$ 10.5A. But this does not mean that this zoom lens has magnification that is invariant to focus change, because zoom lenses usually change focus by complex movements of some of the lens components. To achieve constant magnification in such cases, a special mechanism must be added to shift the relative position between the zoom lens and CCD sensor.

5 EXPERIMENTS

5.1 Magnification

To verify the constant-magnification capability of telecentric lenses, we have taken a series of images by changing the focus level and measured the magnification. To detect magnification change between images, the following method was used:

1) **FFT-phase-based local shift detection:** We compute the Fourier transform of corresponding small areas in the two images and find the ratio of the two resulting spectra. Then a plane is fitted to the phases of the ratio of the spectra. The gradient of the estimated plane is nothing but the image shift vector. As the two image areas should contain the same scene areas to get sub-pixel accuracy, the area used for FFT computation is refined iteratively by using the computed image shift. The image window used to compute local shifts has 64×64 pixels.
2) **Object pattern:** A crisp and dense scene pattern is designed to ensure the phase estimates are accurate. The pattern we have used is shown in Fig. 5b. The period of the pattern must be larger than the FFT computation area to avoid phase ambiguities.
3) **Needle diagram of local shifts:** A sparse needle map (5×4 needles) of the local shift vectors are chosen for display over the 640×480 pixel image.
4) **Magnification change and translation:** A shift vector (Δx, Δy) at image coordinate (x, y), where x and y are measured from the center point of the image, is modeled as a similarity transformation: $\Delta x = a_x + mx$, $\Delta y = a_y + my$. Least-mean-square fitting is used to estimate the parameters m and (a_x, a_y). This way, we separate out the global magnification change m from the global translation (a_x, a_y). The translation factor (a_x, a_y) is introduced in the above model for two reasons:
   (a) the optical center of the image is assumed to be unknown and
   (b) the optical center itself can shift between images due to the shift that results from any possible misalignment or wobble in the focus adjustment mechanism.
   The residual error of the above fit reveals the local shift detection accuracy and the validity of the above transformation model.

Fig. 5 and Fig. 6 show the magnification change for the f = 25 mm lens without and with the telecentric aperture, respectively. In each of these two figures, (a) is the image focused at infinity, (b) at 787 mm from the lens, and (c) at 330 mm from the lens. Figures (d) and (e) are needle diagrams that show the local shifts of image (b) relative to image (a) and that of image (c) relative to image (a), respectively. The needles are magnified by a factor of five for clarity of display.

In the same manner, images were taken and processed for the 35 mm, 55 mm, and 85 mm lenses, and the results are summarized in Table 2. As is clear from the table, magnification variations in the telecentric versions of the lenses are remarkably small. Pixel shifts due to large focus setting changes are as small as 0.5 pixel. With careful tuning of the aperture position, this number can be further reduced to 0.1 pixel. Small translations are observed in the cases of the 35 mm and 55 mm lenses. This is because the focus is changed using the focus ring of the lens. For the smaller lens, the translation caused by the wobbling of the lens is larger, as expected. But such translations can be easily compensated for by shifting the image without any harmful side effects. This is because, telecentricity holds the magnification constant and hence the shifts that need to be applied are pure translations. In the case of the 25 mm lens, which is mounted on a precise micrometer stage (see Fig. 4b), translation between images is only of the order of 0.5 pixel.

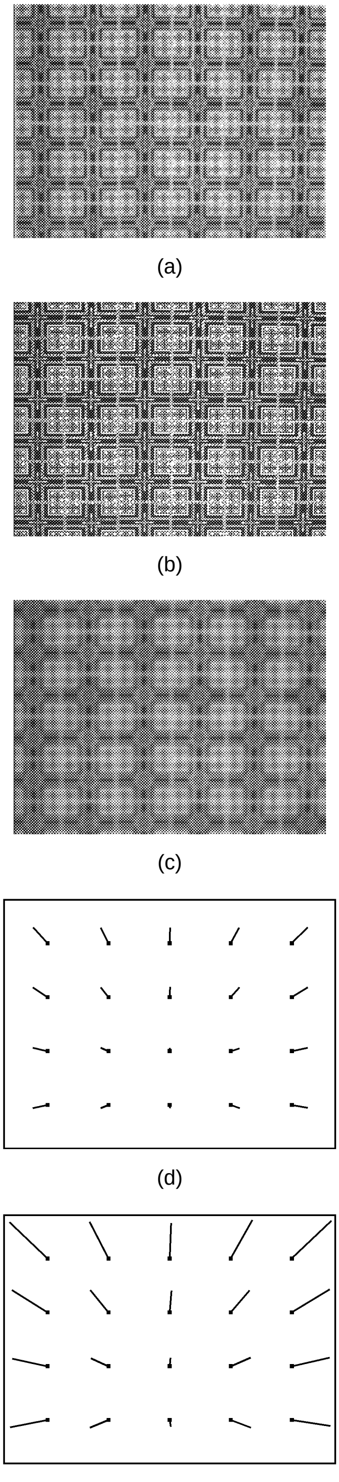
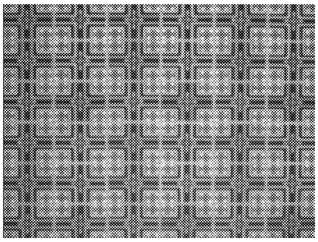
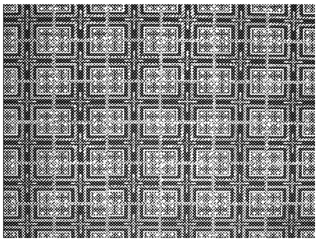


Fig. 5. Magnification variations in a nontelecentric lens due to focus change. (a) d = infinity. (b) d = 787 mm. (c) d = 330 mm. (d) Shift between (b) and (a) $\times$ 5. (e) Shift between (c) and (a) $\times$ 5.



(a)



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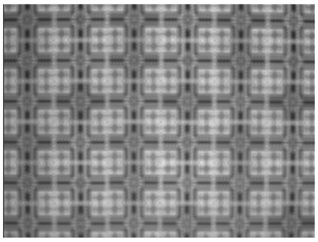
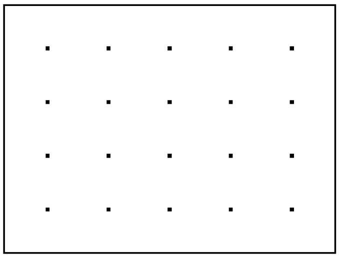
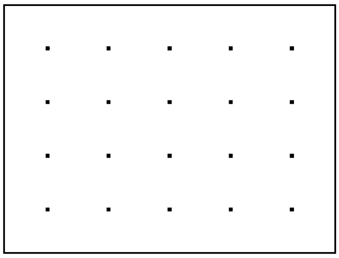


(c)



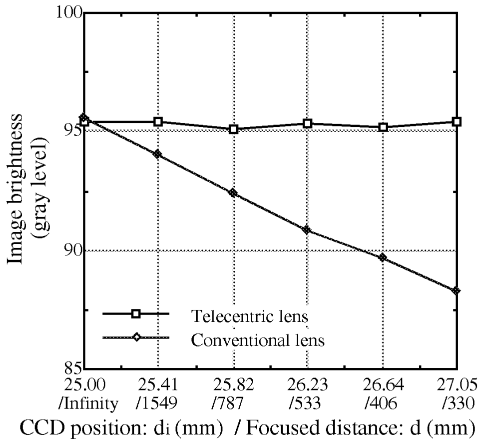
(d)



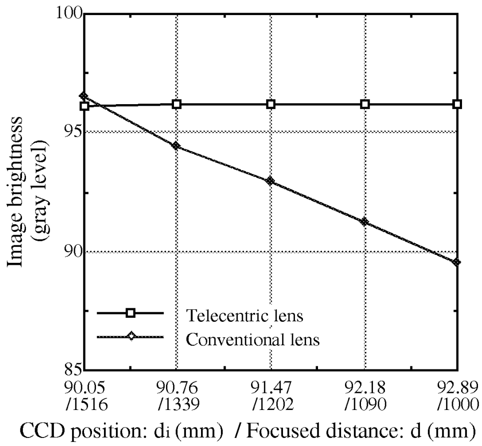
(e)

Fig. 6. Magnification variations due to focus change in the telecentric version of the lens used in Fig. 5. The needle maps in the two figures can be compared to see that magnification remains unaffected by focus change. (a) d = infinity. (b) d = 787 mm. (c) d = 330 mm. (d) Shift between (b) and (a) × 5. (e) Shift between (c) and (a) × 5.

These experiments relate to the geometric properties of both conventional and telecentric lenses. Fig. 7 shows the brightness variation due to focus change for Cosmicar 25 mm lens and Nikkor 85 mm lens. As explained in Section 2.3, image brightness varies with focus change in the case of the conventional lens while it remains constant in the telecentric case due to the invariance of the effective F-number (see (1)). As seen from the plot, brightness variations in the conventional lens are significant. This requires brightness values for different focus settings to be normalized prior to further processing. This process is avoided in the telecentric case. Table 3 summarizes our experiments on the photometric properties of telecentric lenses.



(a)



(b)

Fig. 7. Image brightness variation due to focus change. Brightness does not vary in telecentric lenses. (a) Cosmicar 25 mm. (b) Nikkor 85 mm.

6 CONCLUSION

Most commercial lenses can be turned into telecentric ones by simply attaching an additional aperture. The procedure for aperture placement and the photometric and geometric properties of telecentric lenses were discussed in detail. We have demonstrated that magnification changes can be reduced to as low as 0.03 percent, i.e., maximum magnification induced shift of 0.1 pixel, by the proposed aperture-placing method. The only drawback of this optics is that the F-number that does not cause vignetting is larger (aperture must be smaller) than in the lens prior to conversion. This needs to be compensated by using either brighter illumination or a more sensitive image sensor. Alternatively, as described in Section 4, field lenses can be added to commercial ones to make them telecentric without increase in F-number, i.e., without reduction in brightness.

TABLE 2
MAGNIFICATION VARIATIONS FOR FOUR WIDELY USED LENSES AND THEIR TELECENTRIC VERSIONS

Lens	Shift of CCD*	Change in focused distance†	Nontelecentric		Telecentric	
			Mag.(max. shift‡)	Translation	Mag.(max. shift‡)	Translation
Cosmicar 25 mm	2.05 mm	∞ – 330 mm	5.9% (18.9 pix)	(0.4, 5.8)	–0.03% (0.1 pix)	(0.5, 0.2)
AF Nikkor 35 mm	2.08 mm	4022 – 550 mm	4.5% (14.3 pix)	(1.4, 4.7)	–0.15% (0.5 pix)	(–0.7, 3.3)
Micro-Nikkor 55 mm	2.94 mm	∞ – 1085 mm	5.8% (18.6 pix)	(–1.9, 0.0)	–0.07% (0.2 pix)	(–0.2, 0.1)
Nikkor 85 mm	2.85 mm	1514 – 1000 mm	4.7% (14.9 pix)	(–0.9, –1.8)	0.01% (0.03 pix)	(0.4, –1.0)

* *Measured on image side.*
† *Measured on object side.*
‡ *max. shift is the maximum shift caused by the magnification change.*

TABLE 3
BRIGHTNESS VARIATION DUE TO FOCUS CHANGE FOR FOUR OFF-THE-SHELF LENSES AND THEIR TELECENTRIC VERSIONS

Lens	Shift of CCD*	Change in focused distance†	Brightness variation	
			Non-telecentric	Telecentric
Cosmicar 25 mm	2.05 mm	∞ – 330 mm	–7.6%	0.0%
AF Nikkor 35 mm	2.08 mm	4022 – 550 mm	–5.1%	0.7%
Micro-Nikkor 55 mm	2.94 mm	∞ – 1085 mm	–9.5%	0.6%
Nikkor 85 mm	2.85 mm	1514 – 1000 mm	–7.2%	0.1%

* *Measured on image side.*
† *Measured on object side.*

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