## Supplementary Technical Report for Paper ID 0714

In this technical report, we provide derivations supporting the content in the paper submission titled "Fibonacci exposure bracketing for high dynamic range imaging". There are two sections:

- Section 1 provides derivation of the normalized intensity errors (for conventional and generalized registration) due to non-linear camera response. This was discussed in Section 5, between lines 463-484 of the main paper.
- Section 2 shows a comparison for evaluating the effect of different sensor bit-depths on the results.

## 1 Derivation of the Average Intensity Difference Due to Non-linear Camera Response

Several digital sensors have non-linear intensity response, especially most consumer point-and-shoot cameras, cell-phone cameras and web-cams. Some typical camera responses are shown in Figure 1 (a). In this section, we compare the effect of non-linearities in camera response on conventional and Fibonacci bracketing based registration.

Consider two exposure bracketed frames  $f_i$  and  $f_{i+1}$ , with exposures  $e_i$  and  $e_{i+1}$ . Let I be the image irradiance of a scene point P. The corresponding image intensities in the two images are:

$$E_i = C(I) \tag{1}$$

$$E_{i+1} = C(RI) \tag{2}$$

where C(.) is the camera's response curve, and  $R = \frac{e_{i+1}}{e_i}$  is the ratio of exposures. Optical flow techniques assume that the intensity of a scene point in two images being registered remains the same (after scaling by the exposure times). However, because of the non-linear response, the intensities in the two images may be different, leading to registration errors. The normalized difference in intensities for a scene point with irradiance I, a sensor with response curve Cand the exposure ratio R is given as:

$$D_{conv}(C,I,R) = \frac{|RC(I) - C(RI)|}{RC(I)}$$
(3)



Figure 1: Robustness of Fibonacci bracketing to non-linear camera response functions. (a) 20 response functions (from [1]). (b) Average intensity difference between pixels in source and target images for each response function. Differences for Fibonacci bracketing based generalized registration is always less than that of conventional registration.

where the subscript conv refers to conventional registration. Clearly, the large the difference, the higher the probability of registration error. In order to evaluate the effect of different response functions, we compute the average error over the exposure ratio and the irradiance:

$$D_{conv}(C) = \int_{R} \int_{I} \frac{|RC(I) - C(RI)|}{RC(I)} dR dI.$$
(4)

For a given response curve C, the above integral is computed numerically. The limits of I are between 0 and 1, and the limits of R are between 2 and 16 (corresponding to exponential bracketing schemes with different growth factors). The maroon-colored bars in Figure 1 (b) illustrate  $D_{conv}(C)$  for 20 different response functions. Strongly non-linear curves result in large intensity difference, thus increasing the probability of registration errors.

In Fibonacci bracketing and generalized registration, flow is computed between a frame and sum of two previous frames. Let the three exposures be e,  $R_{fib} \times e$  and  $R_{fib}^2 \times e$ , where  $R_{fib} = \frac{1+\sqrt{5}}{2}$  is the approximate ratio between consecutive exposures. Optical flow is computed between the sum of first two frames and the third frame. The normalized difference in intensities for a scene point with irradiance I is given as:

$$D_{fib}(C,I) = \frac{|C(I) + C(R_{fib}I) - C(R_{fib}^2I)|}{C(I) + C(R_{fib}I)}.$$
(5)

The average error for a response function G is given as:

$$D_{fib}(C) = \int_{I} \frac{|C(I) + C(R_{fib}I) - C(R_{fib}^2I)|}{C(I) + C(R_{fib}I)} dI.$$
(6)

The blue-colored bars in Figure 1 (b) illustrate  $D_{fib}(C)$ , and are significantly smaller than the bars for conventional registration. This makes Fibonacci bracketing highly robust to non-linear camera response functions. Thus, with the proposed approach, it is possible to achieve high-quality results without calibrating the camera's response curve. This is especially useful in exposure-fusion based approaches, where exposure bracketed images are directly merged into a high-quality image without computing an intermediate HDR image [3].

## 2 Evaluating the Effect of Sensor Bit-Depth

The M310 camera that we used for our experiments has a 14 bits sensor. Cheap sensors, especially cell-phone sensors have lower bit-depths (10 bits). In order to evaluate the results for different bit-depths, we emulated an 10 bits camera by clipping the lower 4 bits from the captured images. The resulting HDR reconstructions for different schemes are shown in Figure 2. The performances of both the burst and the alternating schemes degrade significantly as the bit-depth decreases. This is because both these schemes capture only low to moderate dynamic range. Fibonacci bracketing maintains high signal-to-noise-ratio even for low bit-depths, and thus can be used with low-quality sensors. For more results, please see the supplementary image gallery.

## References

- Michael D. Grossberg and Shree K. Nayar. What is the space of camera response functions? In *Proc. IEEE CVPR*, 2003.
- [2] Sing Bing Kang, Matthew Uyttendaele, Simon Winder, and Richard Szeliski. High dynamic range video. ACM Trans. Graph., 22(3), 2003.
- [3] Tom Mertens, Jan Kautz, and Frank Van Reeth. Exposure fusion. In Proc. Pacific Graphics, 2007.
- [4] Li Zhang, Alok Deshpande, and Xin Chen. Denoising versus deblurring: HDR techniques using moving cameras. In *Proc. IEEE CVPR*, 2010.



Burst scheme [4] Alternating scheme [2]

Fibonacci scheme [This paper]

Figure 2: Evaluating the effect of sensor bit-depth. HDR results of different bracketing schemes using sensors with bit-depths of 14 bits (top) and 10 bits (bottom). Performance of the burst and the alternating schemes degrades significantly as the bit-depth decreases. Fibonacci bracketing maintains high signal-to-noise-ratio even for low bit-depths, and thus can be used with cheap sensors.