Synchronous Reactive Systems

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Outline

- Synchronous Reactive Systems
- Heterogeneity and Ptolemy
- Semantics of the SR Domain
- Scheduling the SR Domain

Reactive Embedded Systems

- Run at the speed of their environment
- *When* as important as *what*
- Concurrency for controlling the real world
- Determinism desired
- Limited resources (e.g., memory)
- Discrete-valued, time-varying
- Examples:
  - Systems with user interfaces
    - Digital Watches
    - CD Players
  - Real-time controllers
    - Anti-lock braking systems
    - Industrial process controllers

The Digital Approach

Why do we build digital systems?

- Voltage noise is unavoidable
- Discretization plus non-linearity can filter out low-level noise completely
- Complex systems becomes predictable and controllable
- Incredibly successful engineering practice

The Synchronous Approach

Idea: Use the same trick to filter out “time noise.”

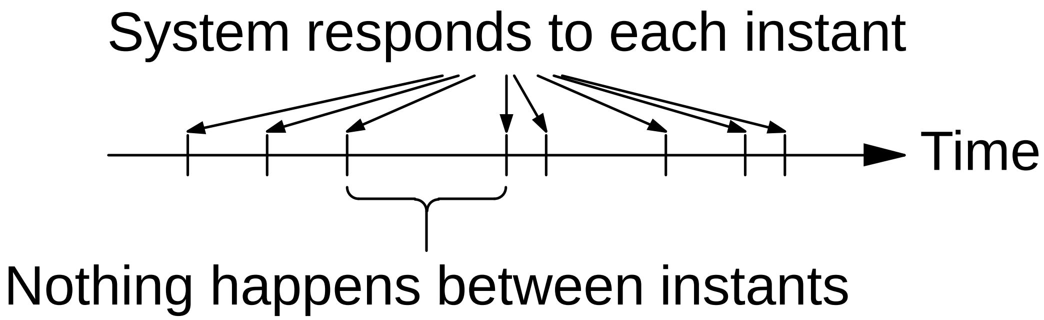
- Noise: Uncontrollable and unpredictable delays
- Discretization ⇔ global synchronization
- The synchrony hypothesis:

Things compute instantaneously

- Already widespread:
  - Synchronous digital systems
  - Finite-state machines

The Synchronous Model of Time

- Synchronous: time is an ordered sequence of instants
- Reactive: Instants initiated by environmental events

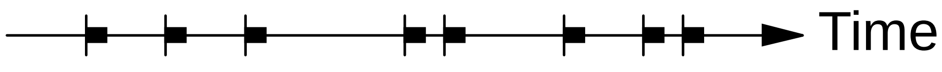


System responds to each instant

Time

Nothing happens between instants

- A system only needs to be "fast enough" to simulate synchronous behavior



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Heterogeneity

Why are there so many system description languages?

- Want a succinct description for *my* system.
- “Let the language fit the problem”

Bigger systems have more diverse problems; use a fitting language for each subproblem.

Want a heterogeneous coordination scheme that allows many languages to communicate.

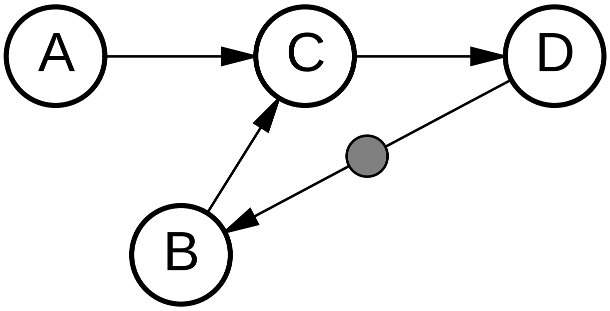
Heterogeneity in Ptolemy

Ptolemy: A system for rapid prototyping of heterogeneous systems

A Ptolemy *domain* (model of computation):

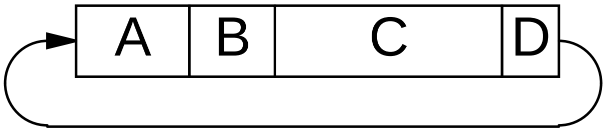
- Set of blocks:

  Atomic pieces of computation that can be "fired" (evaluated).



- Scheduler:

  Determines block firing order before or during system execution.



Schedulers Support Heterogeneity

- Scheduler does not know block contents, only how to fire
- Block contents may be anything
- “Wormhole”: A block in one domain that behaves as a system in another
- Hierarchical heterogeneity: Any system may contain subsystems described in different domains

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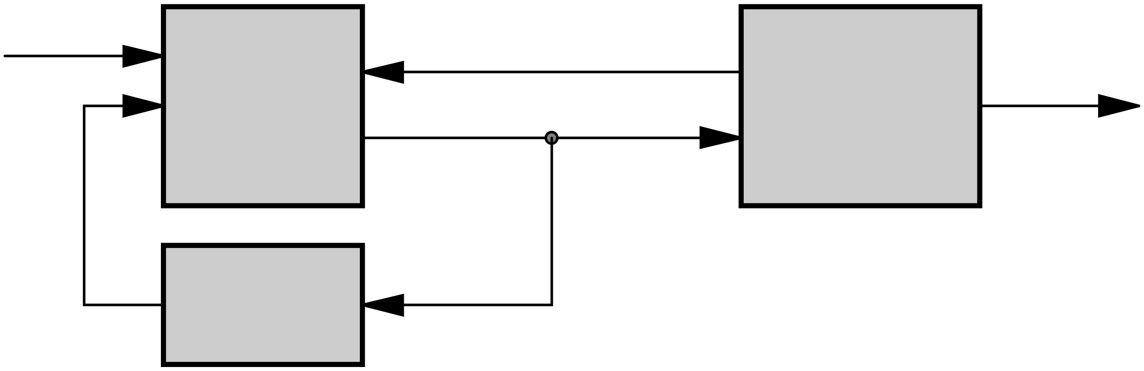
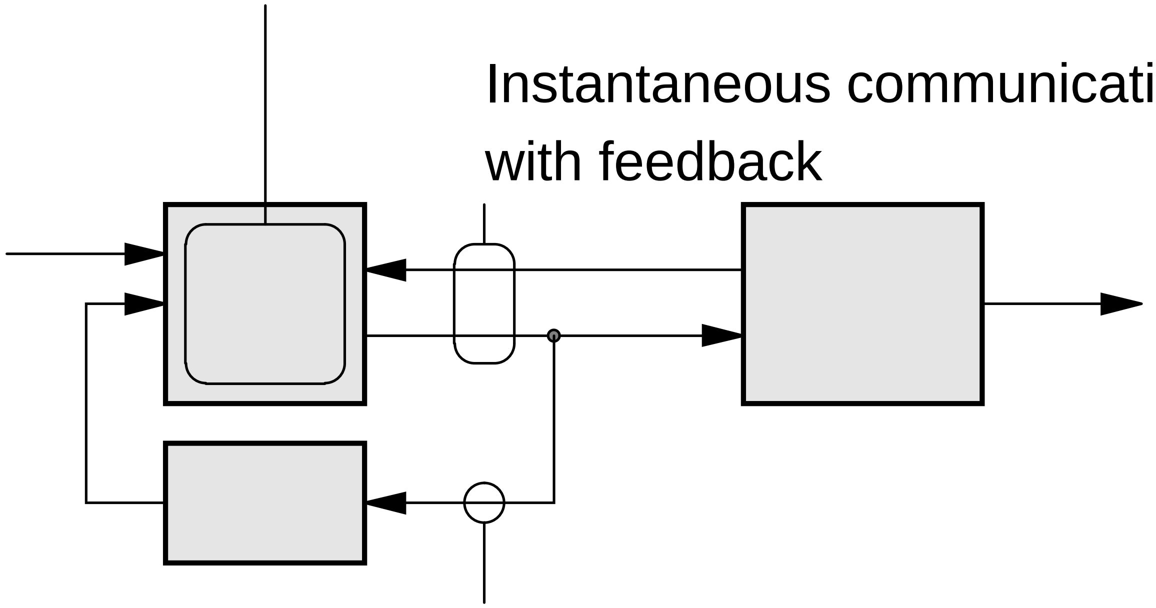
The SR Domain

- Reactive systems need concurrency
- The synchronous model makes for deterministic concurrency
  - No “interleaving” semantics
  - Events are totally-ordered
  - “Before,” “after,” “at the same time” all well-defined and controllable
- Embedded systems need boundedness; dynamic process creation a problem
- SR system: fixed set of synchronized, communicating processes



The SR Domain (2)



Zero-delay blocks

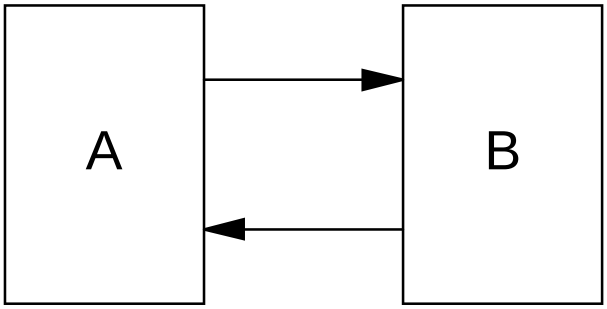
Instantaneous communication with feedback

Single driver, multiple receiver channels

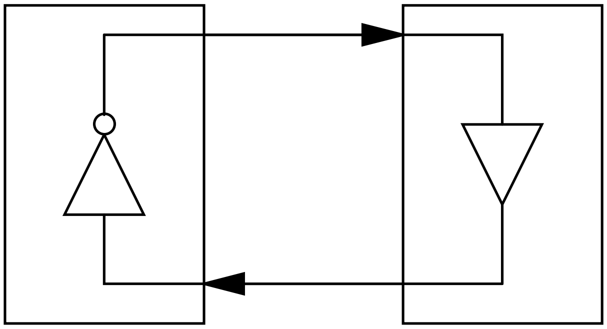
- Block functions may change between instants for time-varying behavior
- Blocks may be specified in any language

Zero Delay and Feedback

How to maintain determinism?



Which goes first?
Need an order-invariant semantics



Contradictory!
Need to attach meaning to such systems.

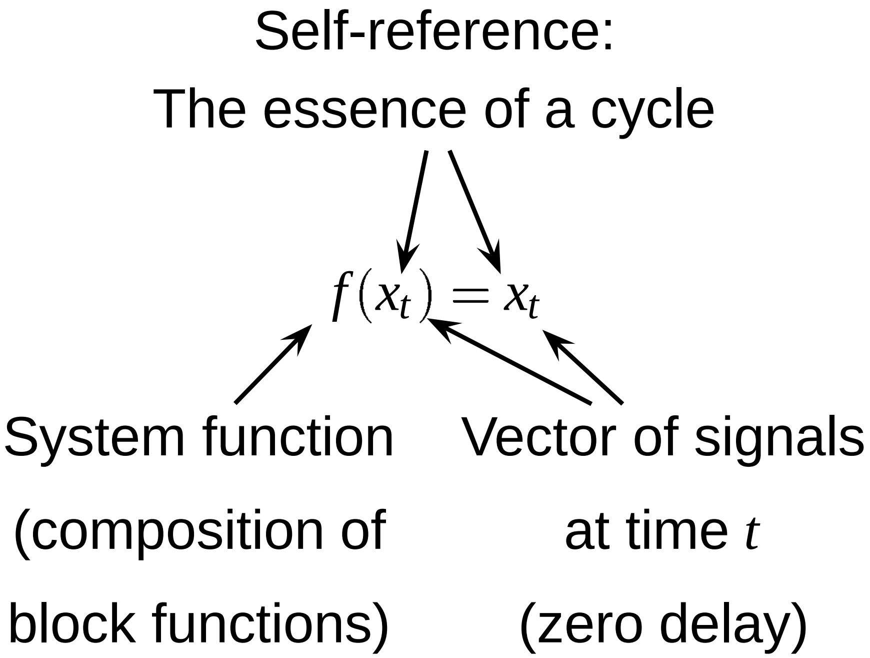
Dealing with Feedback

Why bother at all?
Answer: *Heterogeneity*

- Cycles are usually broken by delay elements *at the lowest level*
- Some schemes insist on this
- False feedback often appears at higher levels
- Data dependent cycles can appear when sharing resources
- *Virtually all cycles are “false,” yet must be dealt with.*

Fixed-point Semantics are Natural for Synchronous Specifications with Feedback

Why a fixed point?



Self-reference:
The essence of a cycle

$$f(x_t) = x_t$$

System function
(composition of
block functions)

Vector of signals
at time t
(zero delay)

fixed point $\Longleftrightarrow$ stable state

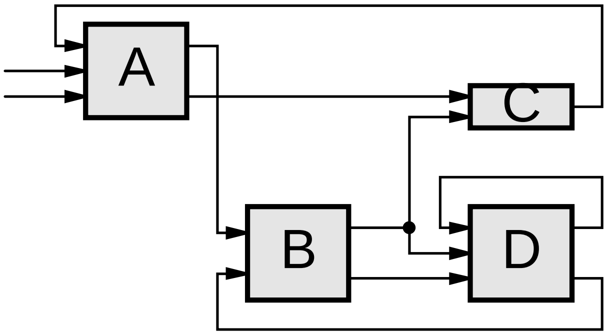
determinism $\Longleftrightarrow$ unique solution

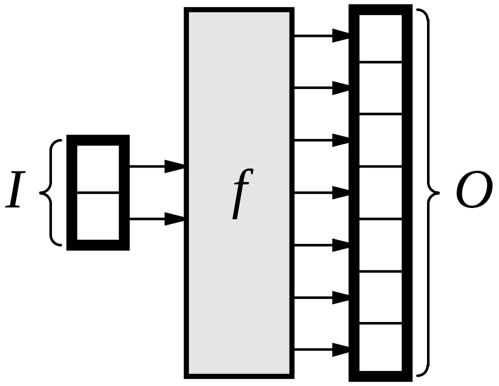
Unique Least Fixed Point Theorem

A monotonic function on a complete partial order (with $\perp$) has a unique least fixed point.

What does it mean to make the system function f monotonic and the signal values a CPO?

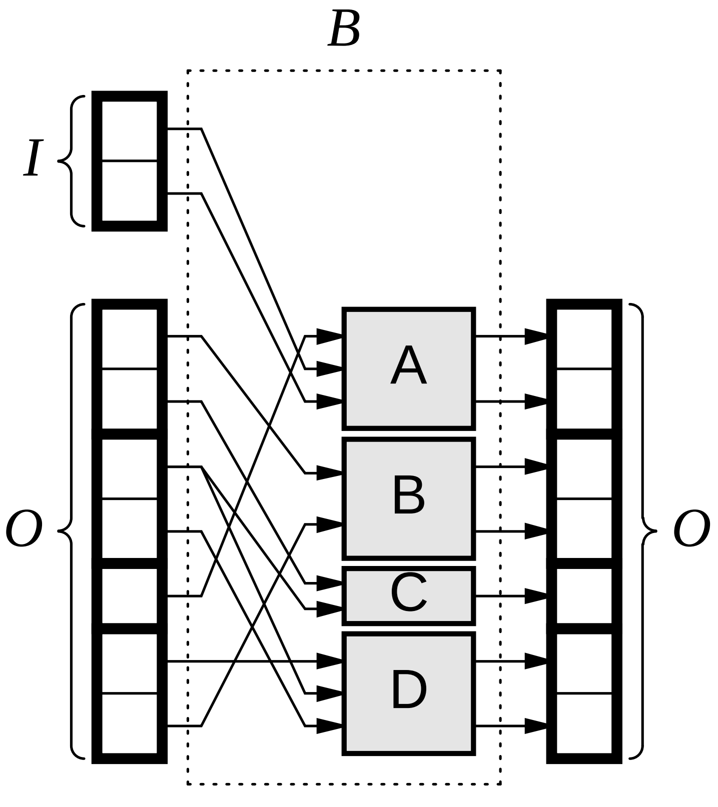
The Least Fixed Point of What?





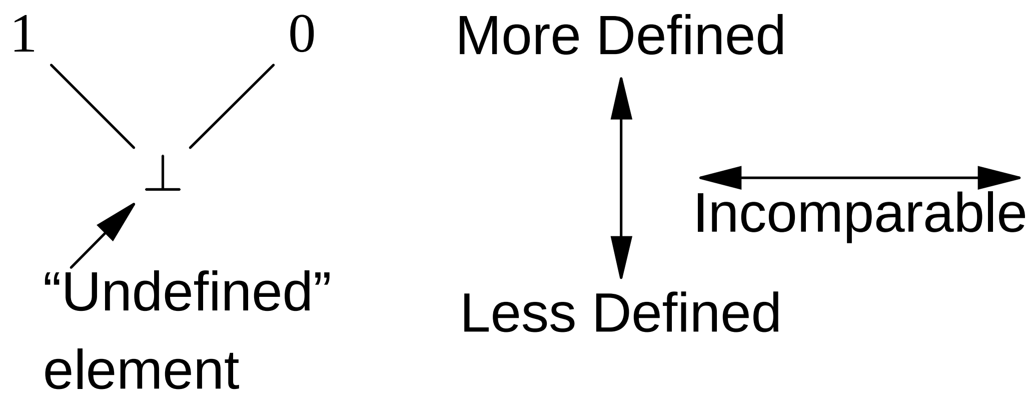
Interpret as ↘ ↗ Take LFP

$$B(I, f(I)) = f(I)$$



Vector of Signals is a CPO

Values along an upward path grow more defined.

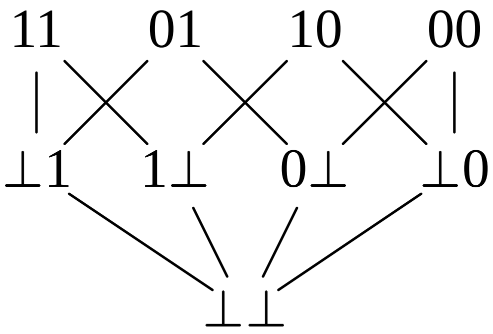


“Undefined” element

More Defined

Less Defined

Incomparable

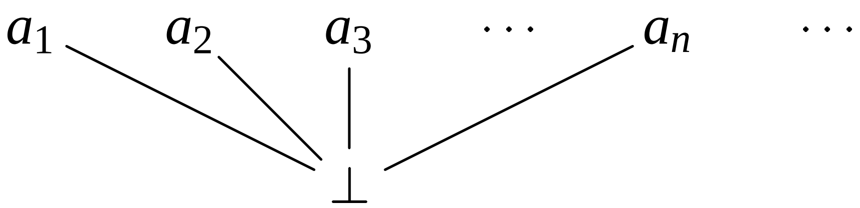


vector-valued extension

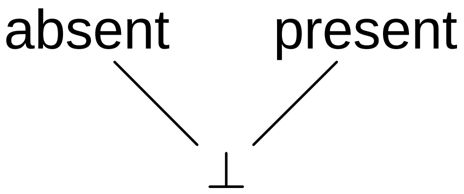
Formally, $x \sqsubseteq y$ if y is at least as defined as x.

Adding $\perp$ Is Enough

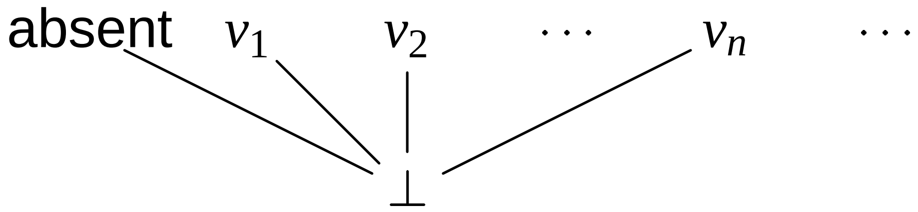
Any set $\{a_1, a_2, \ldots, a_n, \ldots\}$ can easily be “lifted” to give a flat partial order:



A CPO for signals with pure events:



A CPO for valued events:



Why not absent $\sqsubseteq$ present?

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present A then ... else ... end
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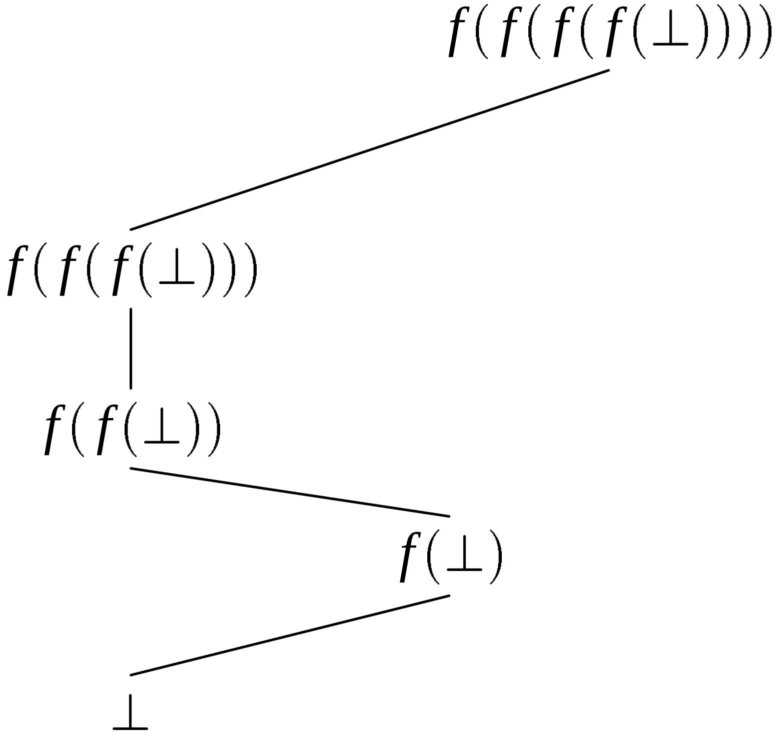
Violates monotonicity

Monotonic Block Functions

Giving a more defined input to a monotonic function always gives a more defined output.



$f(f(f(f(\perp))))$

$f(f(f(\perp)))$

$f(f(\perp))$

$f(\perp)$

$\perp$

Formally, $x \sqsubseteq y$ implies $f(x) \sqsubseteq f(y)$.

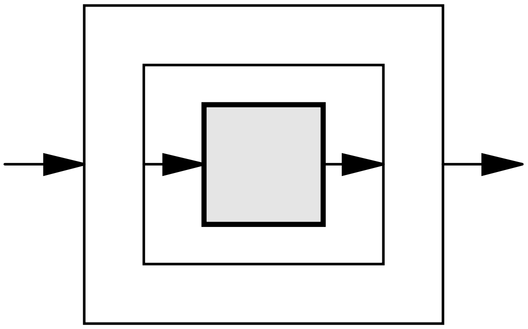
A monotonic function never recants (“changes its mind”).

Many Languages Use Strict Functions, Which Are Monotonic

A strict function:

$$g(\underbrace{\ldots,\bot,\ldots}_{\text{inputs}}) = (\underbrace{\bot,\ldots,\bot}_{\text{outputs}})$$

Outside:
A strict
monotonic
function



Inside:
Simple
“function call”
semantics

Most common imperative languages only compute strict functions.

Danger: *Cycles of strict functions deadlock—fixed point is all ⊥—need some non-strict functions.*

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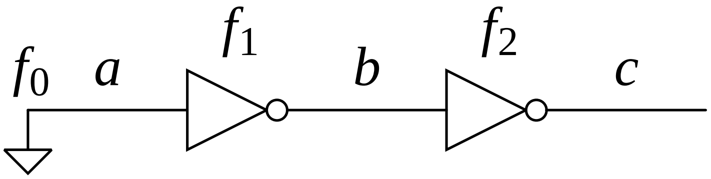
A Simple Way to Find the Least Fixed Point

$$\perp \sqsubseteq f(\perp) \sqsubseteq f(f(\perp)) \sqsubseteq \cdots \sqsubseteq \text{LFP} = \text{LFP} = \cdots$$

For each instant,

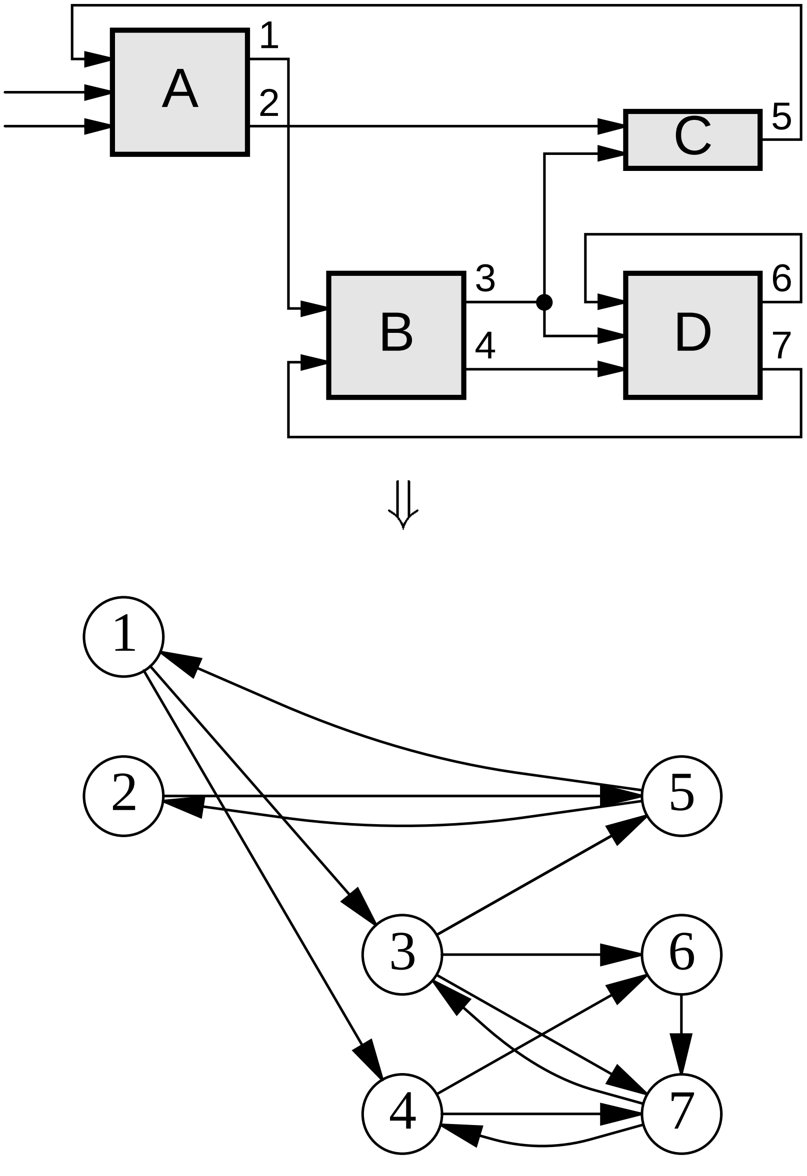
1. Start with all signals at $\perp$
2. Evaluate all blocks (in some order)
3. If any change their outputs, repeat Step 2



$$\begin{aligned}
(a,b,c) &= (\perp,\perp,\perp) \\
f_0(\perp,\perp,\perp) &= (0,\perp,\perp) \\
f_1(0,\perp,\perp) &= (0,1,\perp) \\
f_2(0,1,\perp) &= (0,1,0) \\
f_2(f_1(f_0(0,1,0))) &= (0,1,0)
\end{aligned}$$

The Dependency Graph

Transform into single-output functions:

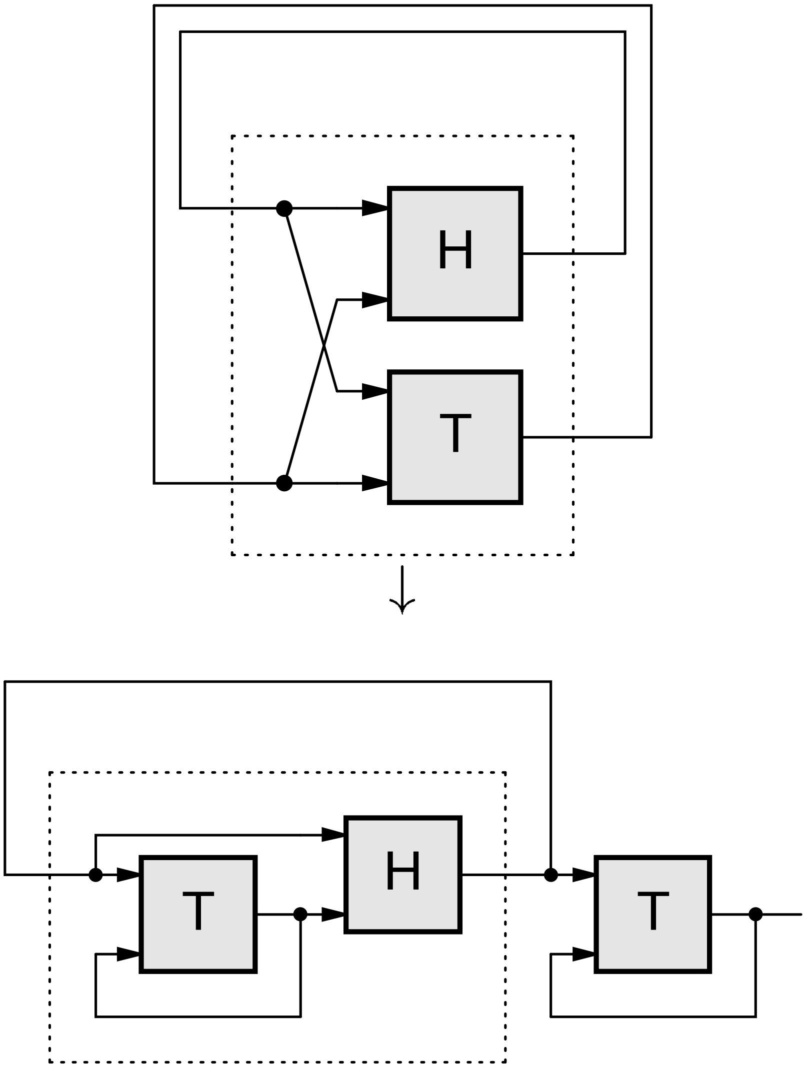


The Scheduling Algorithm

1. Decompose into strongly-connected components
2. Remove a head (set of vertices) from each SCC, leaving a tail
3. Recurse on each tail

Evaluating SCCs

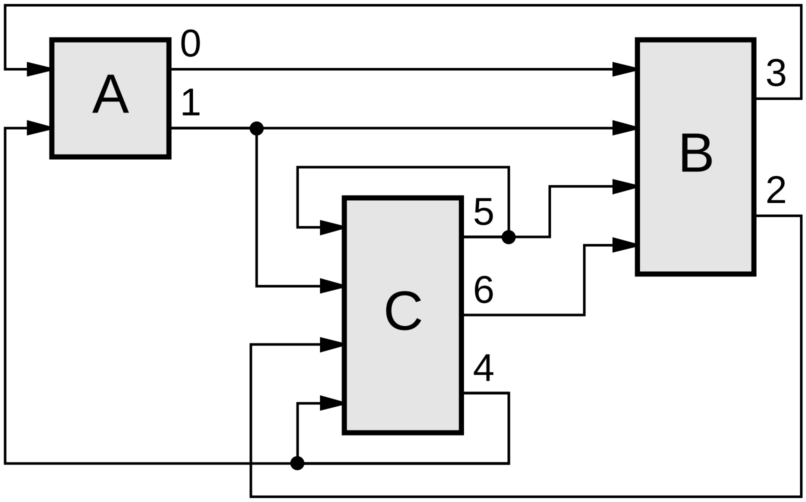
Split a strongly-connected graph into a head and tail:



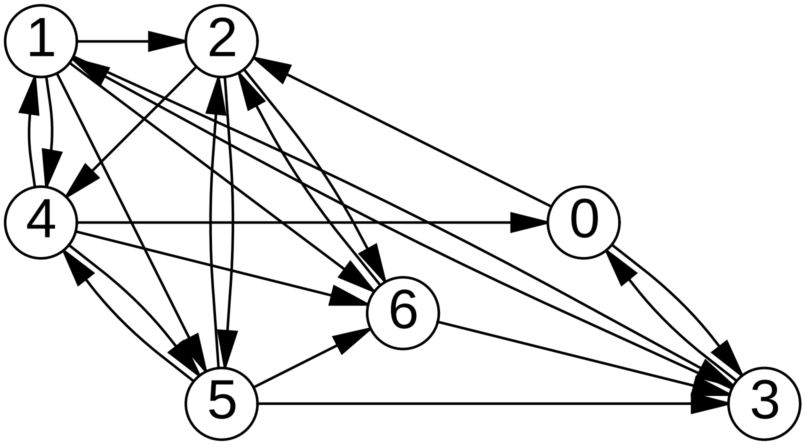
Good heads break T’s strong connectivity.

Example

System

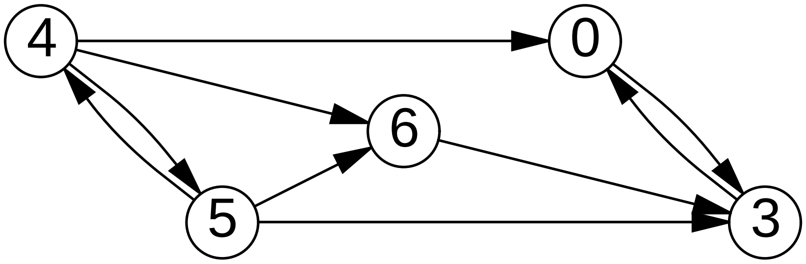


Graph

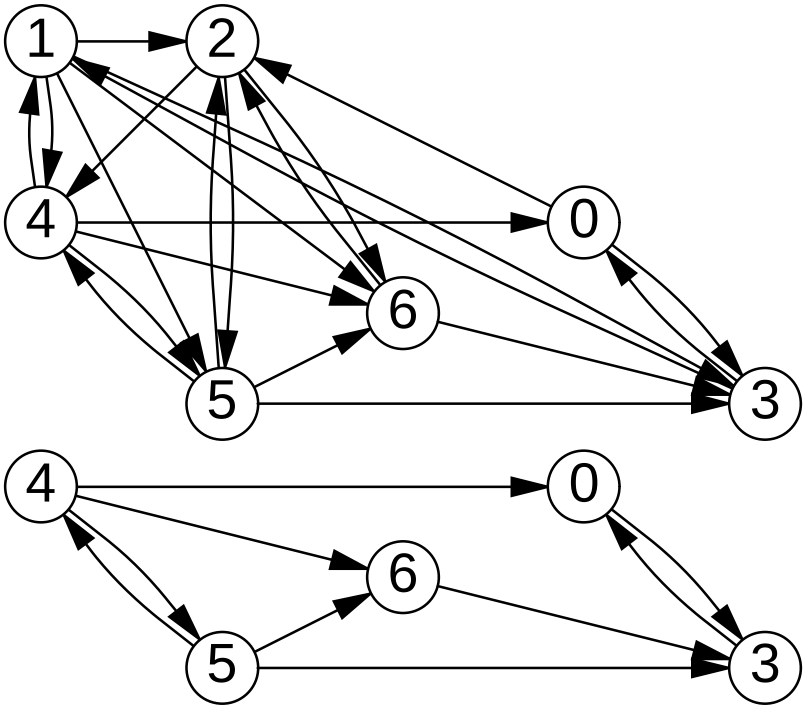


Head

Tail



Schedules

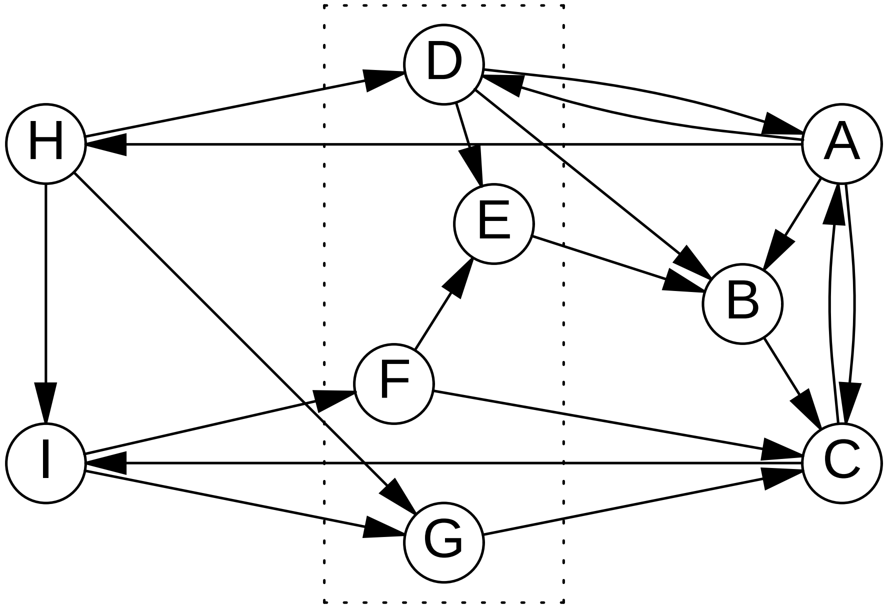


$$(\ \overbrace{1\ 2}^{\text{head}} \ . \ \overbrace{\underbrace{(\overbrace{4}^{\text{head}} \ . \ \overbrace{5}^{\text{tail}})}_{\text{SCC}} \ 6 \ \underbrace{(\overbrace{0}^{\text{head}} \ . \ \overbrace{3}^{\text{tail}})}_{\text{SCC}}}^{\text{tail}} \ )$$

5 4 5 6 3 0 3 1 2 5 4 5 6 3 0 3 1 2 5 4 5 6 3 0 3

Finding Good Heads

Must break strong connectivity—remove a border of a set of vertices:

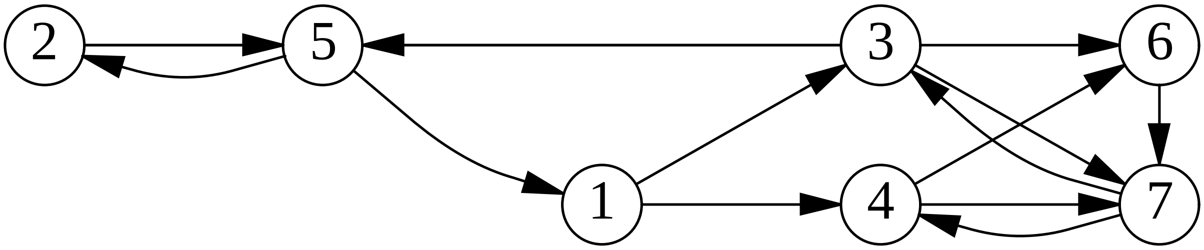


border of { A, B, C }

(vertices with incoming edges)

Choosing Good Border Sets

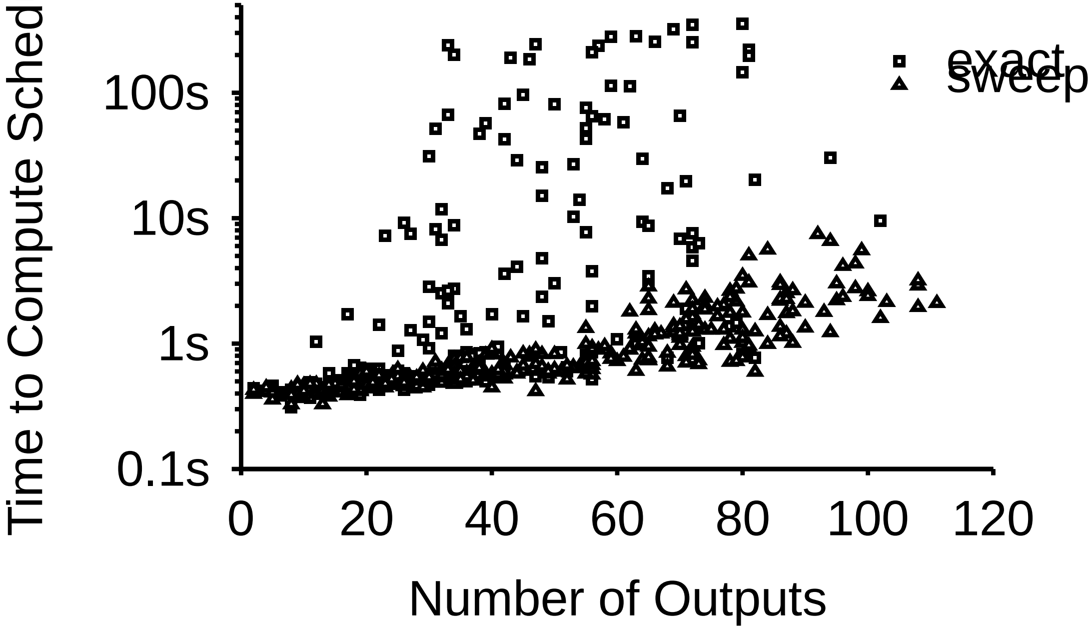
Heuristic: “Grow” a set starting from a vertex and greedily include the best border vertex:

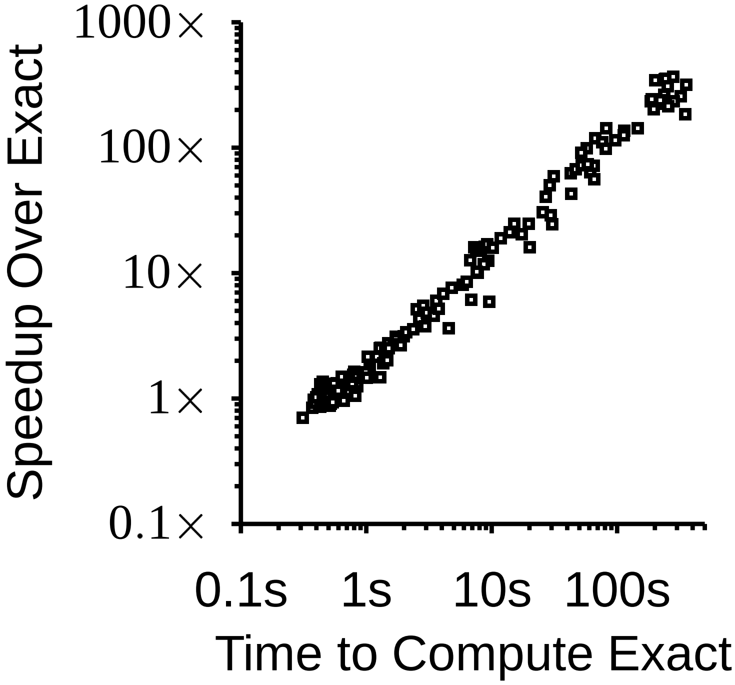


Set	Border
1	5
1 5	2 3
1 5 2	3
1 5 2 3	7
1 5 2 3 7	4 6
1 5 2 3 7 4	6

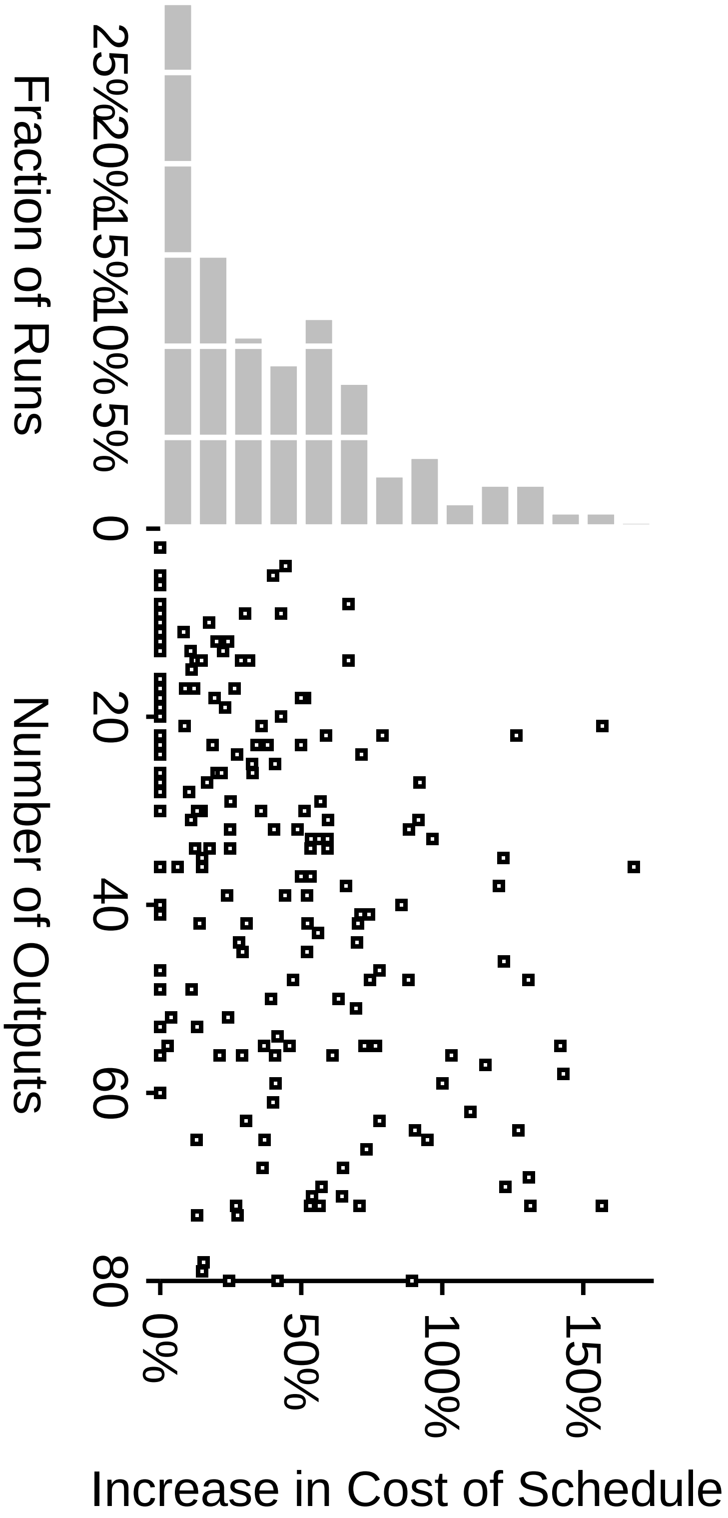
2 is better (3 would increase border)

Scheduling Results

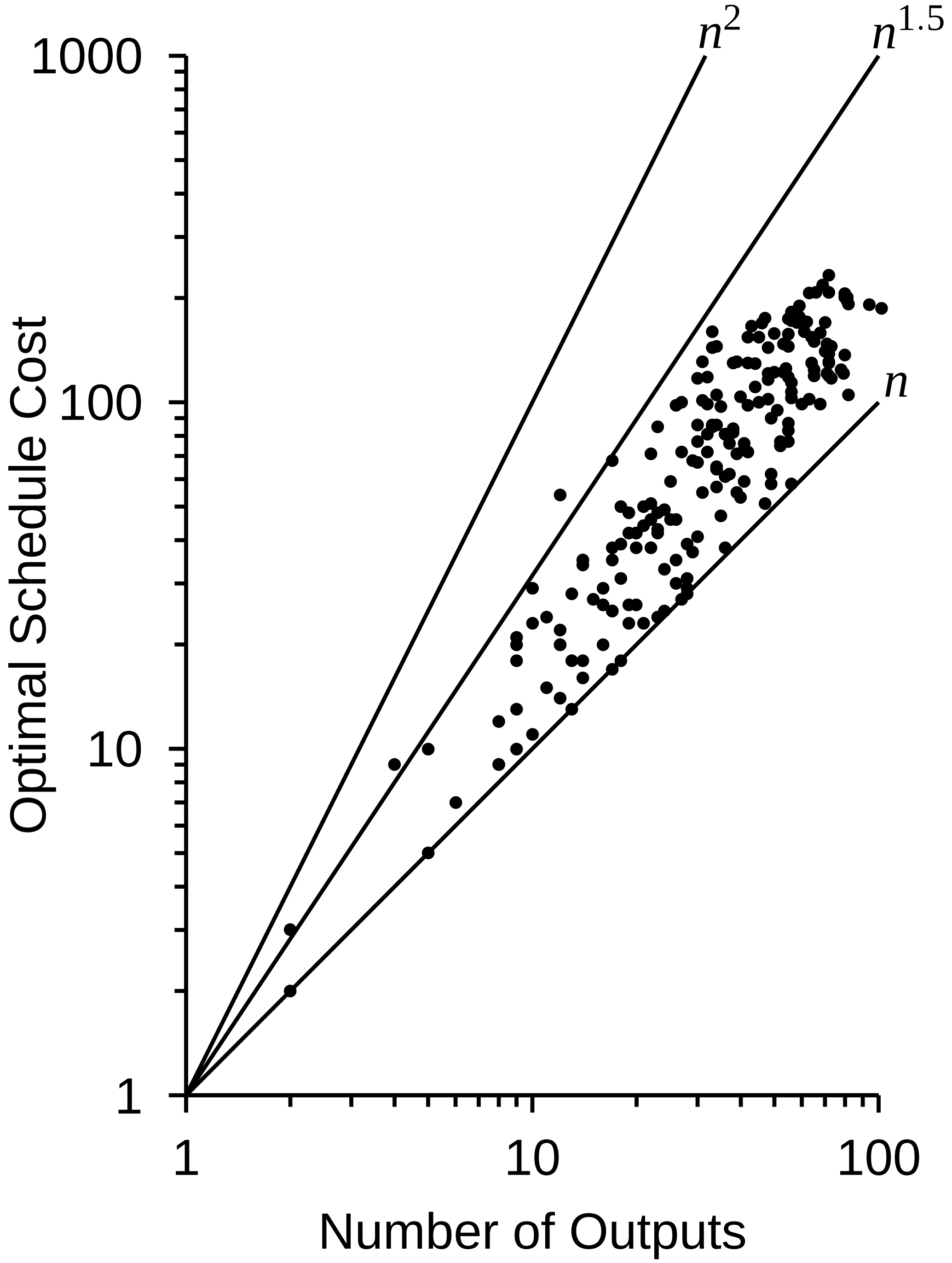




The Cost of Using the Heuristic



Asymptotic Schedule Cost



Conclusions

- Reactive embedded systems
    - Run at the speed of their environment
    - *When* as important as *what*
    - Concurrent, deterministic, bounded, discrete-valued
- The synchronous approach
    - Discrete instants, globally synchronized
    - Assumes instantaneous computation
- Heterogeneity in Ptolemy
    - Domain: Blocks and Scheduler
    - Hierarchical heterogeneity through domain embedding

Conclusions (2)

- The SR domain
  - Concurrent zero-delay blocks
  - Semantics: the least fixed point of a monotonic function on a CPO
  - Values include “undefined” ($\perp$)
- Scheduling the SR Domain
  - Use single-output dependency graph
  - Decompose into SCCs; remove a head from each; recurse
  - Head is the border of the tail
  - Choose a head by greedily growing a set of vertices
  - Fast, efficient. $O(n^{1.25})$ execution