Parallel Cryptanalysis

A Study of the Vigenère Cipher in Haskell

Final Project: Parallel Functional Programming

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Introduction

This paper presents a parallelized Vigenère cipher solver in Haskell. As Marlow's parallelized Haskell RSA encoder and decoders[1] have shown, the process of parallelizing encryption and decryption in Haskell is a trivial task, meaning that there is not an inherent aspect of the algorithm that can be accelerated by leveraging multiple cores. Alternatively, the process of parallelizing cryptanalysis, the formal technique of breaking encryption schemes, often relies on algorithms which do, in fact, have aspects that can be parallelized. For certain ciphers, including the Vigenère cipher, algorithms to search for the encryption key underlying a ciphertext behave akin to graph algorithms, which parallelize rather well.

[1] http://www.cs.columbia.edu/~sedwards/classes/2022/4995-fall/strategies.pdf

Viginère Cipher

The Viginère cipher was created in the 16th century as a successor to the Caesar cipher, which was among the earliest substitution ciphers in history. As techniques to break the Caesar cipher, including early forms of frequency analysis, began to rise in popularity, the puzzling Viginère cipher remained unbreakable for over 300 years after originally being published. The fundamental premise of the Viginère cipher is a ***repeating keyword***. Quite literally, plaintext messages are encrypted by performing an XOR with a chosen keyword that repeats as long as the plaintext message. More formally, the algorithm can be represented as follows: [2]

ALGORITHM

INPUT: Plain text m [ ]

Key e [ ]

OUTPUT: Cipher text c [ ]

Step1 $i = 0;$
Step2 $n = \text{length}(m);$
Step3 $c = 00;$
Step4 for $i \leqslant n$ do;
Step5 $c[i] = \text{m}[i]\ \hat{}\ e\,[i \bmod n] \bmod 26;$
Step6 end for;
Step7 return c.

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[2] https://link.springer.com/article/10.1007/s40305-020-00320-x

In the context of this project, we will consider a cryptanalytic attack on the cipher to be a modified version of ciphertext only attack. The COA[3] setting is where a malicious actor can only access a ciphertext yet must deduce the encryption key used to encrypt the ciphertext. The modifications to COA made in this project are that we will be responsible for both creating and breaking ciphertexts, meaning that we know the correct encryption keys. However, this knowledge is not used during the process of analyzing ciphertexts and outputting a deduced key. Instead, this knowledge is only used to verify that our solver's output is, in fact, the original correct encryption key.

There exists a number of known solutions to finding the encryption key for a Vigenère cipher provided only with a ciphertext [4]. One such solution, the Kasiski Test [5], relies on the fact that identical plaintext separated by the length of the key will map to identical ciphertext. However, this approach is rather fickle and inelegant, as it relies on brute force frequency analysis rather than probabilistic or graphical approaches. The second popularized solution, which is the technique implemented in this project, is the Index of Coincidence algorithm [6]. Relying on the distribution of letter usage in an alphabet, this algorithm will first deduce the length of the encryption key by comparing potential alphabet distributions according to key size (limited to 20 for this project). These steps of calculating distributions for key length do not depend on any other such calculations by key length, meaning that it can be parallelized. After determining an optimal key length, the algorithm then will output an encryption key in order to maintain the letter frequency distribution for a given alphabet.

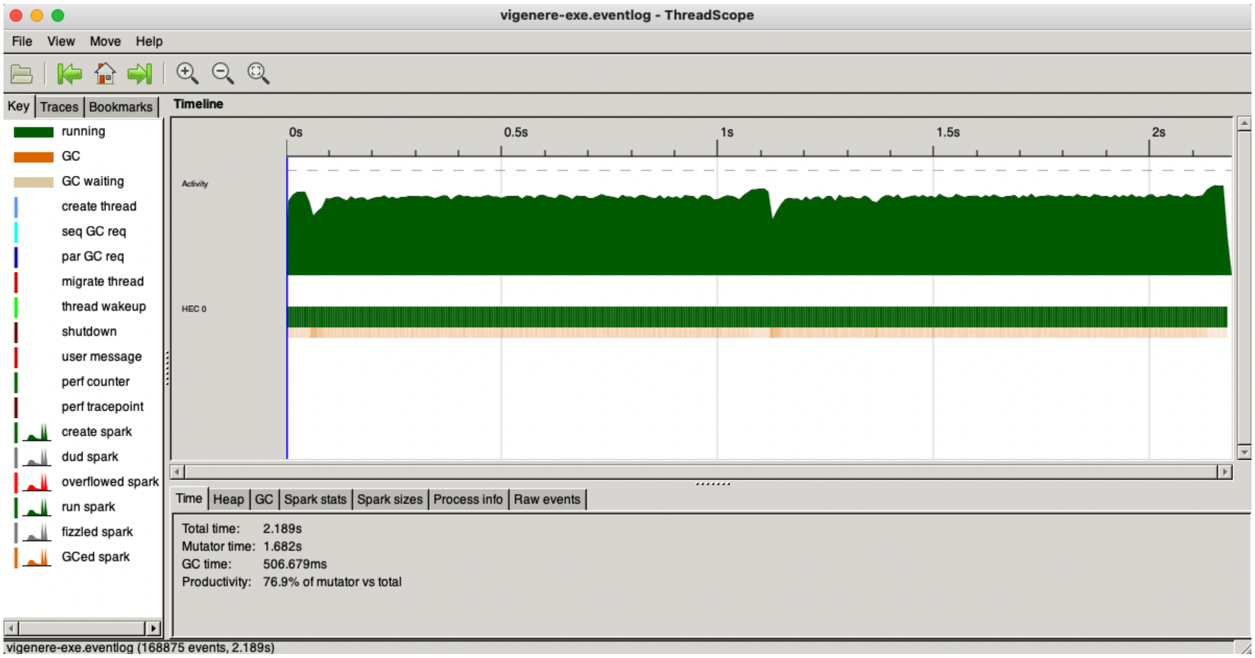
[3] https://en.wikipedia.org/wiki/Ciphertext-only_attack
[4] https://www.cs.purdue.edu/homes/ninghui/courses/Fall05/lectures/355_Fall05_lect04.pdf
[5] https://en.wikipedia.org/wiki/Kasiski_examination
[6] https://en.wikipedia.org/wiki/Index_of_coincidence

Haskell Solver

The solver implementation in haskell is contained in two files, Main.hs and Cipher.hs, both of which are inside the vigenere project directory. Main.hs contains the 3 cipher texts being studied in this project and Cipher.hs contains the sequential and parallel algorithms. The output of the solver algorithm can be printed in the terminal console. Below is one such example of the algorithm's output, where the key and decrypted text are outputted. This example is a paragraph from the Columbia University wikipedia page encrypted under the key "Columbia", which the solver is able to successfully return even though it was provided only with the encrypted ciphertext of the below paragraph. Running **make test** inside the vigenere directory will also output this result.

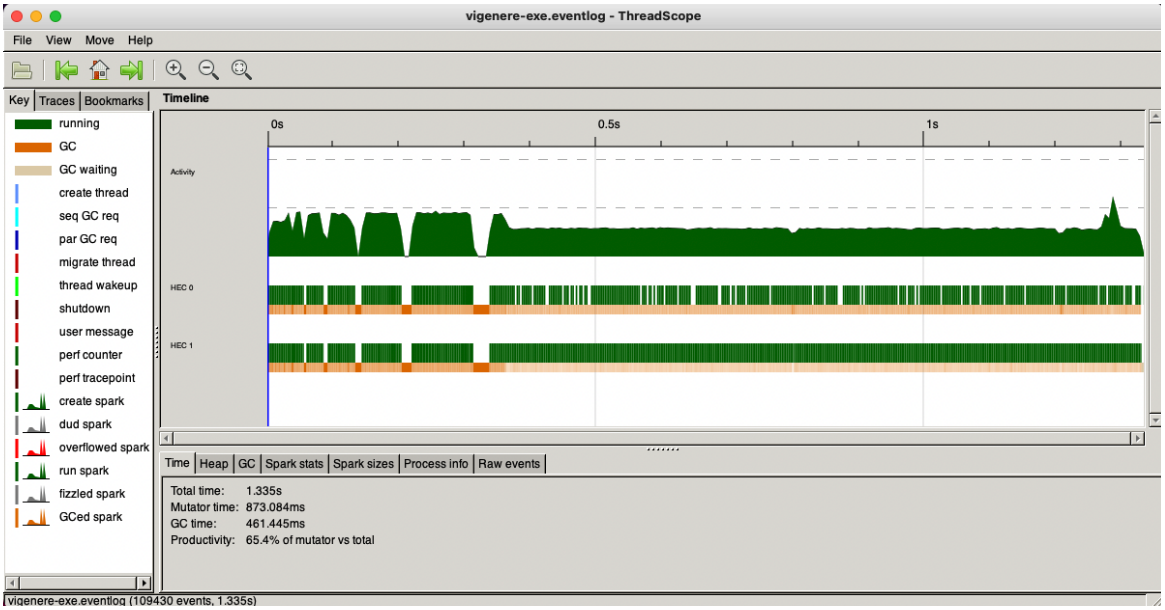
```
Key: COLUMBIA
Decrypted Text:
COLUMBIAUNIVERSITYALSOKNOWNASCOLUMBIAANDOFFICIALLYASCOLUMBIAUNIVERSITYINTHECITYOFN
EWYORKISAPRIVATEIVYLEAGUERESEARCHUNIVERSITYINNEWYORKCITYESTABLISHEDINASKINGSCOLLE
GEONTHEGROUNDSOFTRINITYCHURCHINMANHATTANCOLUMBIAISTHEOLDESTINSTITUTIONOFHIGHERED
UCATIONINNEWYORKANDTHEFIFTHOLDESTINSTITUTIONOFHIGHERLEARNINGINTHEUNITEDSTATESITISO
NEOFNINECOLONIALCOLLEGESFOUNDEDPRIORTOTHEDECLARATIONOFINDEPENDENCESEVENOFWHICH
BELONGTOTHEIVYLEAGUECOLUMBIAISRANKEDAMONGTHETOPUNIVERSITIESINTHEWORLDCOLUMBIAWA
SESTABLISHEDBYROYALCHARTERUNDERGEORGEIIOFGREATBRITAINITWASRENAMEDCOLUMBIACOLLEG
EINFOLLOWINGTHEAMERICANREVOLUTIONANDINWASPLACEDUNDERAPRIVATEBOARDOFTRUSTEESHEA
DEDBYFORMERSTUDENTSALEXANDERHAMILTONANDJOHNJAYINTHECAMPUSWASMOVEDTOITSCURRENT
LOCATIONINMORNINGSIDEHEIGHTSANDRENAMEDCOLUMBIAUNIVERSITYCOLUMBIASCIENTISTSANDSCH
OLARSHAVEPLAYEDAPIVOTALROLEINSCIENTIFICBREAKTHROUGHSINCLUDINGBRAINCOMPUTERINTERFA
CETHELASERANDMASERNUCLEARMAGNETICRESONANCETHEFIRSTNUCLEARPILETHEFIRSTNUCLEARFIS
SIONREACTIONINTHEAMERICASTHEFIRSTEVIDENCEFORPLATETECTONICSANDCONTINENTALDRIFTANDM
UCHOFTHEINITIALRESEARCHANDPLANNINGFORTHEMANHATTANPROJECTDURINGWORLDWARIICOLUMBI
AISORGANIZEDINTOTWENTYSCHOOLSINCLUDINGFOURUNDERGRADUATESCHOOLSANDGRADUATESCHO
OLSTHEUNIVERSITYSRESEARCHEFFORTSINCLUDETHELAMONTDOHERTYEARTHOBSERVATORYTHEGOD
DARDINSTITUTEFORSPACESTUDIESANDACCELERATORLABORATORIESWITHBIGTECHFIRMSSUCHASAMA
ZONANDIBMCOLUMBIAISAFOUNDINGMEMBEROFTHEASSOCIATIONOFAMERICANUNIVERSITIESANDWASTH
EFIRSTSCHOOLINTHEUNITEDSTATESTOGRANTTHEMDDEGREETHEUNIVERSITYALSOANNUALLYADMINIST
ERSTHEPULITZERPRIZEWITHOVERMILLIONVOLUMESCOLUMBIAUNIVERSITYLIBRARYISTHETHIRDLARGES
TPRIVATERESEARCHLIBRARYINTHEUNITEDSTATESTHEUNIVERSITYSENDOWMENTSTANDSATBILLIONINAM
ONGTHELARGESTOFANYACADEMICINSTITUTIONASOFDECEMBERITSALUMNIFACULTYANDSTAFFHAVEINC
LUDEDSEVENFOUNDINGFATHERSOFTHEUNITEDSTATESNFOURUSPRESIDENTSNFOREIGNHEADSOFSTAT
ENTWOSECRETARIESGENERALOFTHEUNITEDNATIONSNTENJUSTICESOFTHEUNITEDSTATESSUPREMECO
URTONEOFWHOMCURRENTLYSERVESNOBELLAUREATESNATIONALACADEMYOFSCIENCESMEMBERSLIVI
NGBILLIONAIRESOLYMPICMEDALISTSACADEMYAWARDWINNERSANDPULITZERPRIZERECIPIENTS
```

Results

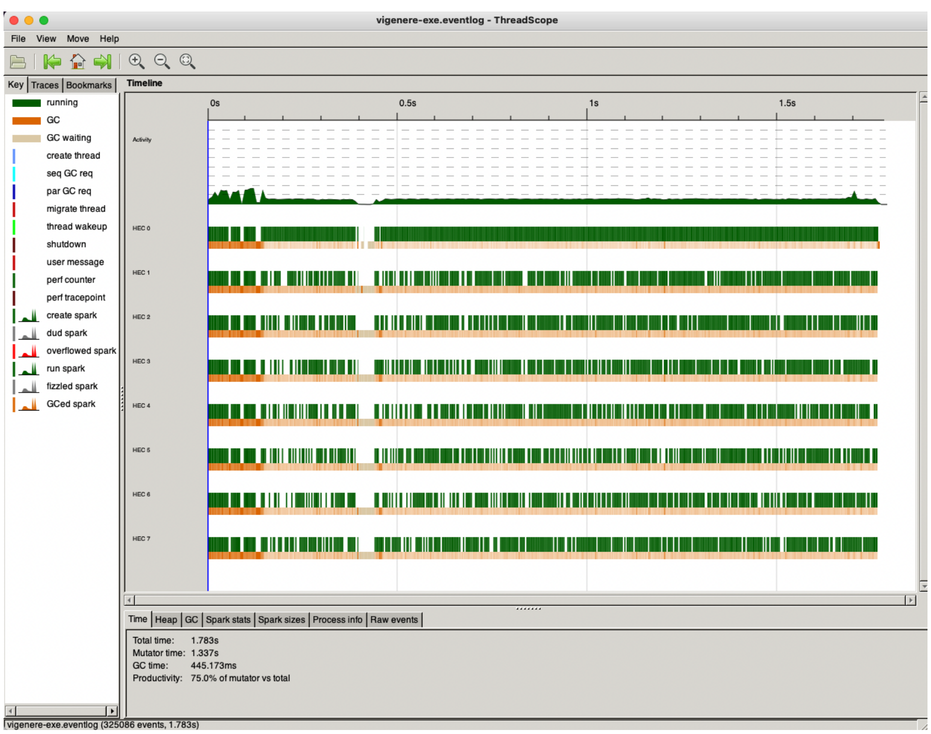


Above is the threadscope result of running the sequential algorithm to break Vigenere ciphers on 3 ciphertexts. The code takes 2.18 seconds to find the encryption keys successfully for each ciphertext, with the 1 CPU running the algorithm maintaining near peak performance throughout the entire process.

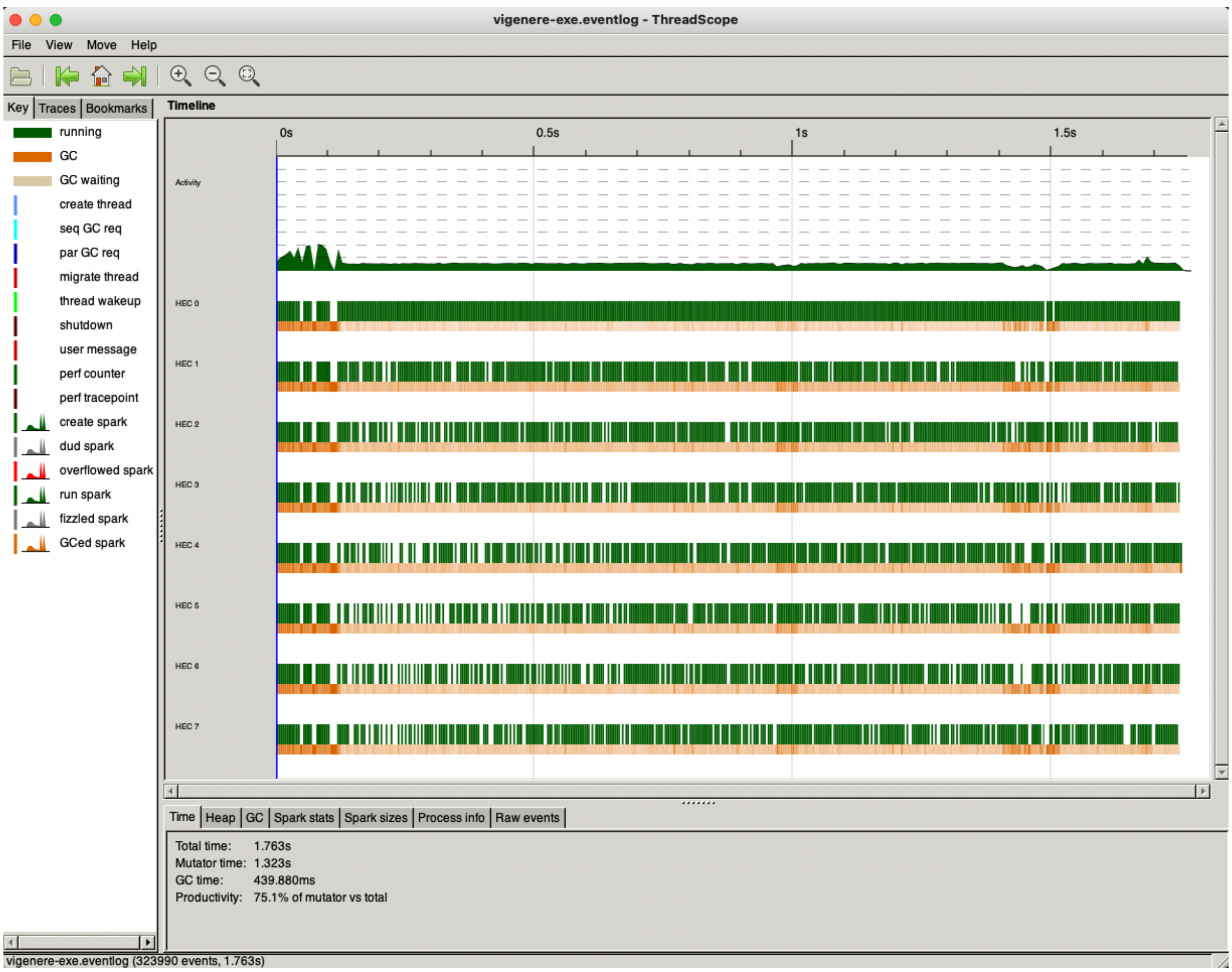
The step of the algorithm which calculates letter distributions for each possible key length can be parallelized. The next step of the algorithm which determines the encryption key after calculating key size can also be parallelizing. Using a combination of **parMap rdeepseq** and **parList rseq** for each of those steps leads to the next result of the project, where a parallelized version of the sequential algorithm is run on 2 cores, leading to an algorithm which runs in 1.34 seconds. This is a 1.6x speedup, with some time lost due to garbage collection introduced when running the algorithm on multiple cores. Below is the threadscope result for running the parallelized algorithm on 2 cores.



However, when this same parallelized algorithm is running on 8 cores, there is a significant amount of sparking. See below for the threadscope results of running the algorithm on 8 cores.

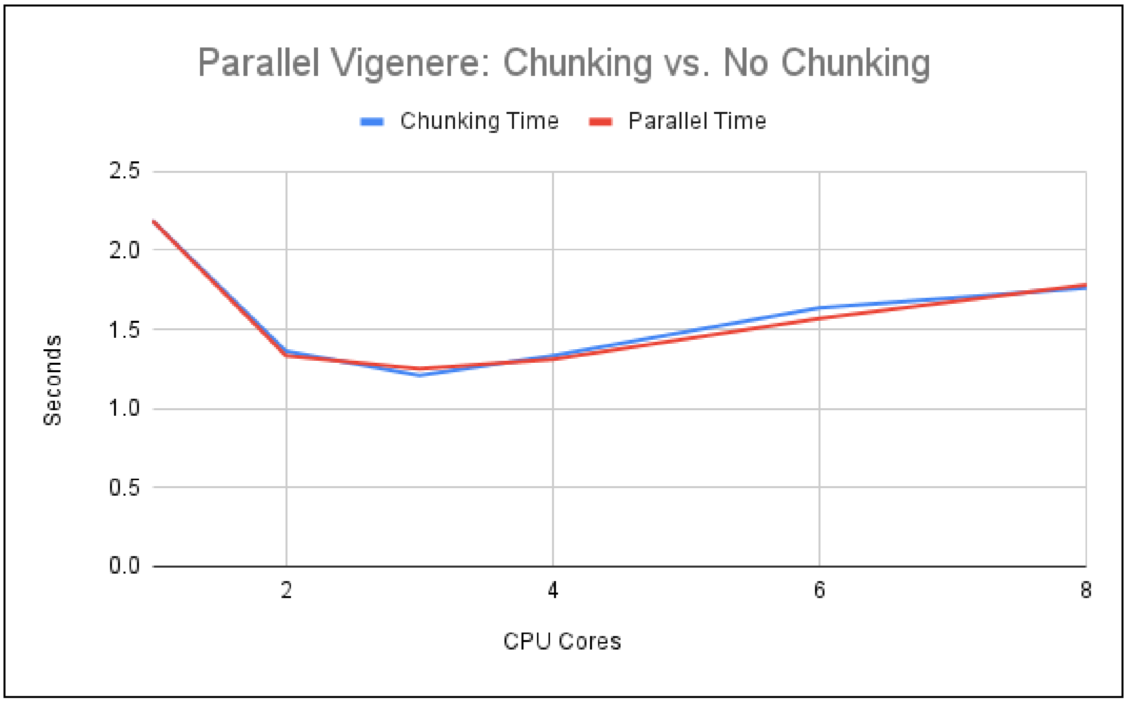


The approach taken in this project to potentially solve this problem with 8 cores is chunking, which can be applied to the **parList rseq** parallelization by instead using **parListChunk 100 rseq** where 100 is the size of the chunk processed by that step in the algorithm. Unfortunately, even the chunking approach did not solve the sparking problem, as is shown in the below threadscope result for parallelized chunking of the Vigenère solver:



While there appears to be a more even load among the 8 cores when utilizing chunking, there is also more garbage collection toward the end of the algorithm, leading to roughly the same results as the non-chunking algorithm.

Overall, a direction for future work in this project is to experiment with additional chunking sizes, which will be further discussed in the conclusion of this document. For now, there was considerable speedup around 3 cores for the parallelized implementations, but then performance plateaus, with a monotonically increasing time to run the algorithm between 4 and 8 cores. These results are visualized below for both the chunking and on-chunking implementations of the solver.



Conclusion

In conclusion, this project has studied the parallelization of the Index of Coincidence algorithm for solving Vigènere ciphers. This algorithm was able to achieve a roughly 2x speedup versus its sequential alternative when run on 3 cores. Beyond 3 cores, significant sparking was experienced, which was slightly alleviated by chunking, but also led to more garbage collection. Future work should explore additional chunking sizes as well as a larger set of ciphertexts to further explore performance results.

Code

Main.hs

```
module Main (main) where

import Cipher (vigenere, parallelVigenere, parallelChunkingVigenere)

main :: IO ()
main = do

   let crypt1 =
"LOEDVSVIDPSXTAMFWILSYECLSZISGIHFQWWAQIEPGBVXZLNDMJTTLRCSJTSWUTREZEVLSOVLEPATILOUXHWGE
NUPANHDRACKEEXWYTRVWTAIKGZLSPMJHXWZXUOWRIFHLACPOLTRVWTQEVXSNASRACXHLRVLRVMYWXNZRYVRSRJ
PMNJIUVNNAAIGHNLWUTRIZJVLPFMYLIWVDRENLWUTRITPXTCIKSOCXSCHWFRLOIBHOXXCYXZHTGMDAFEVVWAHO
WWISFJYFMIBWLLMTTLOOEBILWIPCEULSOGGCHWZXUOUZSFILEYISAHDLAGHWZXUOTRIJTXODIWAWSXZBECSILO
FSVKIAIKGZPCYQWVVAPQSSKSRKTGOEHOPTMLHPHDFGCJAVPKILIIHOPTMLSCSNRPDMASVAHJOLPSUDPSMAMSWE
AYHYZWGXHISMNHDLWUSGRRVMIVXZNEIIIPLUXXKRINVMAHCKQHCIAIJGYROWSAERLQOPTRMFTRTVVVBNMEFBEL
TSDTDYRSAFAZRDLNXSPLMTYELAEXHSCXSDIWAIXKSQPEVHAUGCIJVIAEXVBNMEFLLAKFDVONCEPRIJXZHTRIUP
RRVTGYTKWKTIMVXZIYRMKEPIXLLVFDLWGIVFPLAHORWLISKWLHTOQSAGOCQLOICMKILEJIJNEKRLLLOCMCLAQS
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JFJLAUWGUVODXZHTCTJXRGNLWUCOGGBJOIXKLEWHLDGODIVPSMSEUSRKWOLLVWEPVKBMFNOPWUDXLRRVTABOFD
WOFRWYJEWLXGEYEVDIDLNPPOLVSYMNGGBTECPVAHOWWHOIGTAUGUIJCWTFXJBSDXZTMRYIWSSLYLILEESJDEIE
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```

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RVCAXXSVIOIXZHTGMLWFOKLZLLKFGJVDZRZPSMSMCXRPWOYEMOAZRONRGABEXLGIAJSFZCKTAIELTSFMECWVPR
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DVMXJHKRUTAICPZHVOQWZMNXAZFCREFRIMRCUYOGRETAIKLGBTWCKIMRSEFXUYRWLLOIVGYSMSETYPFRZPMVMC
TSUIWLYAXKWVERDIFASMPWPZEESLAODLWXVMFYDKBEXOXXHKLWHINSXJWEDEUIEDLSHMDVGGTEGLSIGODIEHYD
METENUXZLHYYJGYNJXZYOEKZILEISMNHOWLSEYSEFXUYAGGXHPQSJBOXZLISKEQBPYRQDYRCIAZUBIEPGBVXZN
IFIETCOLVXHVYYJBCDLPDIRKMFLESNVGBGRXOXXHKLAUGCJGGKOKXWUKSRVVINKPWTEXCGJVPRMFZABIJTKIJX
WYDGLWGIEMIJFDKCAIYREXZLLOEXISRVEVAHOQDTXUJXGDABHLWIKZRYAHSRCJTOEAZHTRELWGHRRULDKRVPXM
FVWAIWILWIIEXWYIWLSKMNXAWPGRHAIPEKYKZPOECDYRWVWLHOEJIWERGZAOYXZTVBRRIBOFIJNKLRHDFMKGTT
XHKMDSTRIFTROLKZJOWIXGMEEHKLXOYFIWCVRWPVPSJGISKLWWAVEUTJLFYJPSRIFIIRUYFJAXQSAGOCQVVNKP
TPMNCIFUOHEFSETKIFKAXXKSYNTEFPSOBWRYTZSFKOXIGCGANHGYABIFDXTYSKLIXGGBQIJWAVNIILGITLVFKM
KPUDPMDCDPEQILWIYRVWUODCWIGODITHCUFMIMHRZWZPYOWLMTYSFLTRELHEWYMEKIOAZDHIUVWWOBXLWETMIJ
FFBEFZPYYIUVNPIKHHHZWLYEKWGCWIDTDVROHQDYRYMYONOWKEERUSFHNNWWIJOIXZHDOIHGIPVRLHNMIFDXHZ
RYPNRMKAMFVFWJAWIZXQLZOWAHOPWPZIEKAAHOHATHAJSFLTRELWEDSIWUSDYVXIDZRZPSNISILTFXZYOGEOPC
TYIVLABIKIXHZRYOEYAWSESKAWYEKGSGILVWKARSJDTHUEGSUTRIJTWNFEJATYJACHTYIEPNNWUDRSKVMJTSSF
XRTYIXHCOLWLESRKWUTVIEPROEAZVMSFMXPTRRSISYPMIITIYKAEXXWGQATFWAHLEFFYOISKZAXHSCKUJSOVRD
LATWTTSMZIXXZTWIESXTYSRYGETZXMKEOZWCRONASZHOENNSNDILOOEEJIWOWEJIEPSJTXHRXKDIPXWHXWZRYV
FBIUDQPVRKLICWDDATFSNLRDECTXHVIOVUVHLWSUYEVZTVIKHHEJIJCENXZPXTYIHYOZSJIMOEFGAHYJLWENBW
SUDZEQBINKQANHDLSKIBVIFTIXIGCPYZLSCEVIXIXOJEQTOBIAHXHPHMLTREFBSRVXZHNKPDRENGEQTAMFWILT
YIKLRFMUTENUXZLLYCSAXYZSOLIXHGXRGZXHHYCMLHILWCGBRRMYWREJWHHRDMKISRVGWPVOSMGHUKMWZAXHGJ
VDLXALSKVWISYFYJAHBSFTENUWLHTOGZXPDIIFHNNWWGZAEXKDHSGZSSBLXOOADXZTCSYSMSDLCVDMNXINLRIX
ZXRGJEXLTYASGHYFYJSOFISCHHFRGBRNYFRENNIDJOWIZXXHVVAOAFITTKUEXGWLKRLILEVEFKWSPDAEBFYJAO
WECTXHVIXBLVSXVVONMFNNYFDTFAEUMVTRELWESKRGSECWVTWEIZWKNYVEJWTSICUOGRFDPEJWLVHKZWSSNVWG
SEDQWTRFFPVAHOISCHHFPVAHOILDQYYISYTLEFFYOKLWYESJAVVONXZLHKVNTWTZWQVUBSOCHUEGSUMITDTRTV
SMZJYCKLENKSFPNPYDCISJWWLKDSZXHEKLWTSOPNTWIEHJVPCSXHSRISOZOXWCXRSDIFAHKRWHENUCGBWRSKTT
LRGWZABILWINVEJLSDOFDAWVAASLOWLPFLZWZVUBIKIETVYHVNYYJTPDVWLTAVGGAQWYSEDEXEETLEIISMTOVL
WIPIMFJEYJUJQBVVDHNNAZXGHYSFVUBQMHXNFXMUAMGGBTAEMWKIXZWHXHZQGULIFMIWIXRKVFXSTAINVWKSIU

```
IKIERJWZHLVWZXREFRSSLNIKTVVVVKMRYQZTRCVXGPNFIJCISJEFKBSRVJWFLVLOEBXGNSUDEUIEDLLWIRVWLP
SVETDYRNLAJHSWFDXUJIVMOBCGJMLCFWTYCIDUXHVLSYBSRYTVAEHEHKONGNJUCXZLHOEJXRGFJEFWSJWLMTYC
GBRKTHGSATLKVHEQTACTROWTYVISKIDLRUHNWCODVTYCUHWNSJBECSILOACMVTXHVTJPNMIGUGUDFWYLKRVILA
KMKHSDIHDRWYM"

   putStrLn ""
   vigenere crypt1
   putStrLn ""

   let crypt2 =
"BXNTSTLQMXTHWCFMKXSZFJWGQTMYFKMTCVVDKTUVEBVTWCZTBXWGNWVRQIROSVGTBHDSAROBWKFMPUTRHFWZP
VXROZGZBHCBWGIMKYWBUPCMICQAHBATCCTIIYKBIADQTNWVFUQMWHQBOLBIWBNMABIBIGVZXUDZBWQWGDIEUWY
VVMFPTNVWWACCMVVMZBQGDSVRGQMUOZRMWLVWNNUZNUHMQUPBTRXNSBRKGAGJTETSYHJZXFHWCSMOGBBQPCUNS
ACFVWKBOJFXKQDWFFILQZCGJWGVCBUFLHWPTRTXXPRQAHXKQPTRNCLKBONQMXTHWCFMKPSBJPZDVVMAFBPQFSG
JUXUHIZQAMTOVFBKMKCVFCGACGPVOOMJSUVOBHCBWAHWBPUKUBQGQTPNTPUCGMQQZHQTWSXWKMTWENQGIOZRDW
KFHPNUKTPBWGCMVJOVTFLPKHPBVBKGRWVOOMJSXEPWYQTEBSSMJSTBOOXUHKUBQGPCBBOTRUSZIFATUDZBPNHH
HPRTMJWSVPFWYGJMAUAPKHVRTAXFPCGQZHQTBUBBBVQIZFNKQABUFTTTUMFUXHQZWSDXNRCERSILNCVTBATOOR
BSQMACNPQCIQKMEJAVQBBEPTEGRJLOWWGGBUBBTTSVBUKHQDMEBBBPUBBBBMCQSGIMGGHEBSSMJSGYMOXPSZNU
MMJSTBOOXUHKUBQGCBLBVBICQMNUBTEYMETBAGBMGXWKMWBFFTYTSYHJZXUAQAJUTNGBEVKMWFMZFALCUMFBZX
DFWNEKTUHWABJXUHMSGWKVPIFJATPRVBEMLEOVYFIOGOVQSMCQWVGIMGGHEBSSTVKQYMIVESXGJVZVVMYPVZGG
BCSWHHCNJPZDEVIVOILRFWBGWYYVIGIIIRSVREEAKZMGIMRYSZRHWGG"

   putStrLn ""
   vigenere crypt2
   putStrLn ""

   let crypt3 =
"ECWOYCQAWBTPQSAIVMLFEPSNQKYUEDWLWAMCMBVDQTQCOJILNMLMOPTUOPTUGOQVGFDCFZQNVVPWUUGOHBPQK
PZKKGLJDJDAVSTPKMMAIIPLQTMATQSOZJDETGTNKJVNGKJIDLKIVMPMFBJLKGSYPJVAUYTHSTKONZPAQPVTJSR
LAVVDUCQNDJVIVMNBGSKHKBXUZIITVOYWAMCMDWLCEUPEQZOYEUQNUHTNGUQOPCQBUHPETSOOOBBIQBTHZFEYQ
FVUZEBHGTTZFIWLFSDNUOATKHFNUPVOHVTATFZLGOCHUOOIPHSYGOQTGRDNMUMSKHTMAOMOHBTHQDWLQBTUXDW
LNSRYEGWUPRPXBSQOTHZNTFLEEZLLMUQOPCQCZEMPGBOYZDMSGJPHAGEHKQSVQMWNIHZNTFQVAZPUSVMCQZFGN
JIIUFLHWFLAOCYAFIMTQDFHUWMRUWECQTQNVVPQASTDECWOYCQAYODYEUIBNWDBQEJYTCJUXDPATHPLGOLETUP
IDHMIKCQADFITDFTNMJVIVKLMDFVAOSOWAMCMDWLWAMTEISTHRPTLQKTHSUPECAPLUDINTSGIXVBIQBLHPJVWC
GAFMDMDWBOYDBXRKJLNQCWATRZZFSCSVSPMTFIDGRMSRPZMGFDNGEMNVGLFQYINFSCBMNQLVCYUZEROJBUUKJV
TJSNUYQCSYODGAWMDVCTNEDCRTSYNXPKAVWZHUOUOTBTHSTQDGVPCSIBSCBOLQOIMGRNIXVUBKOFHUWMRUWESO
PTUOPTUEDQEPHTMFTINFGNBAMIRUVLPQQTAASOUBJDOVOWLAMMIPGNCQOBIHWNVDFIKVVCIGHPSKBNFGEQNIPC
UUOKOODFNQSQNVSCZMDMTJSWUEFZAPRXUEFZNWQWYMSUAIBPNUDZEUCYUZDMTJSQCDTBNWQWYMSXINSEBQGQRU
HYOOMMATTTMEJWNTSLWFJWNKBEBQBUETWNUEUPEHWCMFFDIFSYWQGWRRZLNQUMCVCYCOTINFQZHFJVEPHLFPSQ
FVOYXYVKHQTEBQJVIVWLFDFAECFNBMOLPNOYHUOOFQFEBQNINJOENMOXRQXPWFECRKBRQASTDYOCCUDWLWAMCM
JAOTULHUAMDKBEIFXMNVMDWTPWLUWYWXVLIPUQIGSCNFSCADBLUCHPMOIWONGLHPHZAFILNQTKHQCWMFIMUPWG
YDTQTAGCYEFIREVPZRPZTUWYWXVLEVVPFMNWNVRZBQSBYGOCNTPJSGFGUFPZYVVPAAELATRTHEUQTWHPZASAPC
```

```
QPMFVLIGGLHPBKCGZPLMUWRNOMIDBBOTWPMIJBHDWRNQDPFKFXMEVKHCGLGMAWNCBOCNNKONIXVUBQSCTZOZEQ
NIAPGNFZOHHSYMTAOEWLNUPVOHOXYDJKAPIYCHFZSKHTYEBVDYODNTFNITGEMOIWONWYNTFCNKHPXEUITGGEIS
SINVHSYYELEIFPYFIMUPWGYDTQTAOWMABVNWOWFKBLMKBTMFFZSVVPJGMQTBSCJDJHEYWEBAWMROWWFUPVVQZF
GQTKONIXVUBCNKJPLEJBYNWMLMSGIUHSYFIQRFZLLSFATRFTPMUMRGGPUDDPLKPCUDZQNVVPOZJBEFGEUFFATJ
SFHUWMRUWESEFVDQKXYZUATCBOMMUJINZTIZJVAOCYAFIMLCFRYEUWFCBJUOBLEOWNCZTBIVIECAOISQTOYOFU
BGFTNEBTUOBTZMDCLVMLHPTBAHTSUHFQNEZFXQEAEXSYZAVVDKBRZMUPETGZZFIMUPWEYPTBAVSDHRPCRWGALQ
TQDGBEMZGWRGWRHTFIDUCQMFBBEPHHIEFKRGHLLUFAGGBPLMMWFVVPOZJBEFBLNUPVSPHPHVVATKQPMAGBHGIY
CFFLSVOEYETCPTSXYOPCRVCYYAGEHQANODSMNVZJMQSDEUBZVQMTAWFPUFFANCHTIZBTAEOOYYZWFUQTYZDMSO
SXVQSALKJTHSCQLNWZHMJZEUCWSYQQCOSOUXJATUONUPFUYCKLLPXQNPSCMMOLPWZTNLFZPTWKYDFKIRWPHFT"

   putStrLn ""
   vigenere crypt3
   putStrLn ""

   putStrLn ""
   parallelVigenere crypt1
   putStrLn ""

   putStrLn ""
   parallelVigenere crypt2
   putStrLn ""

   putStrLn ""
   parallelVigenere crypt3
   putStrLn ""

   putStrLn ""
   parallelChunkingVigenere crypt1
   putStrLn ""

   putStrLn ""
   parallelChunkingVigenere crypt2
   putStrLn ""

   putStrLn ""
   parallelChunkingVigenere crypt3
   putStrLn ""
```

Cipher.hs

```haskell
{-# LANGUAGE TupleSections #-}

module Cipher (
   vigenere,
   parallelVigenere,
   parallelChunkingVigenere
) where

import Data.List(transpose, nub, sort, maximumBy)
import Data.Ord (comparing)
import Data.Char (ord)
import Data.Map (Map, fromListWith, toList, findWithDefault)
import GHC.Conc (par, pseq)
import Control.Parallel.Strategies (
   Strategy,
   evalList,
   using,
   parMap,
   rdeepseq,
   rparWith,
   parListChunk,
   rseq)
import Data.Ratio (denominator)

vigenere :: [Char] -> IO ()
vigenere crypt = do
   let filteredCrypt = filter (/=' ') crypt
       dists = fmap (wrap filteredCrypt) [1..length filteredCrypt `div` 20]
       optimalDist = maximumBy (comparing rate) dists
       key = fmap (deriveCharacter engFreqs) optimalDist
       chars a b = ['A'..'Z'] !! ((ord b - ord a) `mod` 26)
   putStrLn key

parList :: Strategy a -> Strategy [a]
parList strat = evalList (rparWith strat)

parallelVigenere :: [Char] -> IO ()
parallelVigenere crypt = do
   let filteredCrypt = filter (/=' ') crypt
       dists = parMap rdeepseq (wrap filteredCrypt) [1..length filteredCrypt `div` 20]
```

```haskell
      optimalDist = maximumBy (comparing parallelRate) dists
          `using` parList rseq
      key = parMap rdeepseq (deriveCharacter engFreqs) optimalDist
  putStrLn key

parallelChunkingVigenere :: [Char] -> IO ()
parallelChunkingVigenere crypt = do
  let filteredCrypt = filter (/=' ') crypt
      dists = parMap rdeepseq (wrap filteredCrypt) [1..length filteredCrypt `div` 20]
      optimalDist = maximumBy (comparing parallelRate) dists
          `using` parListChunk 100 rseq
      key = parMap rdeepseq (deriveCharacter engFreqs) optimalDist
  putStrLn key

engFreqs :: [Double]
engFreqs = [
  0.081, 0.014, 0.027, 0.042, 0.127, 0.022, 0.020, 0.060, 0.069, 0.001,
  0.007, 0.040, 0.024, 0.067, 0.075, 0.019, 0.000, 0.059, 0.063, 0.090,
  0.027, 0.009, 0.023, 0.001, 0.019, 0.000 ]

aggregate :: Ord a => [a] -> Map a Int
aggregate = fromListWith (+) . fmap (, 1)

avg :: Fractional a => [a] -> a
avg as = sum as / fromIntegral (length as)

parse :: Int -> [a] -> [[a]]
parse _ [] = []
parse n as =
  let (h, r) = splitAt n as
  in h:parse n r

wrap :: [a] -> Int -> [[a]]
wrap as n = transpose $ parse n as

count :: (Ord a, Fractional b) => [a] -> b
count str =
  let charCounts = snd <$> toList (aggregate str)
      l = length str
      denominator = fromIntegral $ l * (l - 1)
      numerator = fromIntegral $ sum $ fmap (\c -> c * (c-1)) charCounts
  in numerator / denominator
```

```haskell
parallelCount :: (Ord a, Fractional b) => [a] -> b
parallelCount str =
   let charCounts = snd <$> toList (aggregate str)
       l = length str
       denominator = fromIntegral $ l * (l - 1)
       numerator = fromIntegral $ sum $ parMap rdeepseq (\c -> c * (c-1)) charCounts
   in numerator / denominator

rate :: (Ord a, Fractional b) => [[a]] -> b
rate d =  avg (fmap count d) - fromIntegral (length d) / 3000.0

parallelRate :: (Ord a, Fractional b) => [[a]] -> b
parallelRate d =  avg (fmap parallelCount d) - fromIntegral (length d) / 3000.0

sumMult :: Num a => [a] -> [a] -> a
sumMult v0 v1 = sum $ zipWith (*) v0 v1

swap :: Num a => [a] -> [a] -> Char -> a
swap v0 v1 letter = sumMult v0 (drop (ord letter - ord 'A') (cycle v1))

deriveCharacter :: RealFrac a => [a] -> String -> Char
deriveCharacter possible sample =
   let charCounts = aggregate sample
       samp c = findWithDefault 0 c charCounts
       real = fmap (fromIntegral . samp) ['A'..'Z']
   in maximumBy (comparing $ swap possible real) ['A'..'Z']
```

Makefile

```
all: vigenere

vigenere:
	stack build

run: vigenere
	stack exec vigenere-exe

test: vigenere
	stack exec -- vigenere-exe -t

threadscope: vigenere
	stack exec -- vigenere-exe -t +RTS -N1 -ls
	threadscope vigenere-exe.eventlog

threadscope2: vigenere
	stack exec -- vigenere-exe -t +RTS -N2 -ls
	threadscope vigenere-exe.eventlog

threadscope3: vigenere
	stack exec -- vigenere-exe -t +RTS -N3 -ls
	threadscope vigenere-exe.eventlog

threadscope4: vigenere
	stack exec -- vigenere-exe -t +RTS -N4 -ls
	threadscope vigenere-exe.eventlog

threadscope6: vigenere
	stack exec -- vigenere-exe -t +RTS -N6 -ls
	threadscope vigenere-exe.eventlog

threadscope8: vigenere
	stack exec -- vigenere-exe -t +RTS -N8 -ls
	threadscope vigenere-exe.eventlog

clean:
	stack clean
```