Parallel N-Puzzle Solver in Haskell

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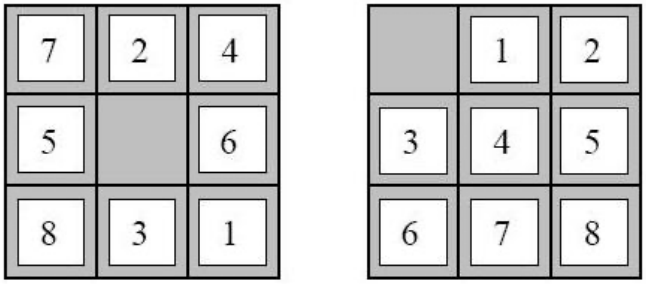
2021/12/22

1. Introduction

N-Puzzle is the general name of category sof puzzle games with $\sqrt{N+1}$ by $\sqrt{N+1}$ frames and *N* sliding tiles inside the frame. For instance, an 8-Puzzle is played with a 3×3 frame and 8 tiles, numbered 1 through 8, placed inside the frame, leaving one empty slot unoccupied. The player is then able to slide adjacent tiles into the unoccupied slot until the *goal state* is reached (e.g., the board is solved). The goal of this game is to turn the scrambled *start state* into the *goal state* in as few moves as possible. The illustration below shows one solvable combination of the *start state* and the *goal state* of an 8-Puzzle game.



2. Solving N-Puzzle

Finding the optimal solutions for N-Puzzles is an NP-Hard search problem. For any generic BFS algorithm, the number of states in its queue quadruples after each iteration, giving it an exponential run time complexity as it searches through $(N+1)!$ different possible states of the frame. According to [1], an optimal 15-Puzzle solver using BFS can reach upwards to a depth of 80 with billions of intermediate frame states. For this reason, our implementable of the N-Puzzle Solver seeks a solution that is fast to compute as opposed to one that has the fewest moves.

3. Implementation

3.1 Algorithm

Our “best first search” N-Puzzle solver uses A* search in combination with Manhattan Distance to conduct informed searches within all possible moves.

Our A* search takes both the state of a next frame and the state of the search space into consideration. In our case, the resulting frame of any valid move is given a score of *g(x) + h(x)*. *g(x)* is the cost, measured using Manhattan Distance, from the *start state* to the potential *next state,* while *h(x)* is the cost, also in Manhattan Distance, from the *next state* to the *goal state*. We then add each *next state* along with its cost to a min-priority queue where A* search chooses the state with the lowest cost from.

3.2 Data Structures and Representations

We are using a 2d-arrary of *Int* to represent the frame (referred to as *grid* in the code)

```
type Grid = [[Int]]
```

We are using a min priority queue for (cost, candidate). Each candidate is an array of G*rid*s from the start to the *next state* of the grid.

```
import qualified Data.PQueue.Prio.Min as PQ
PQ.MinPQueue Int [Grid]
```

We are using a set of grids to mark the states we've already visited to avoid loops.

```
import qualified Data.HashSet as S
S.Set Grid
```

3.3 Sequential Implementation

```
min-pq<- [(start_state, 0)]
while (min-pq is not empty):
    cur_state <- min-heap.pop()
    if (cur_state.state != goal_state):
        for new_state in getNeighbors(cur_state):
            if new_state not in visited:
            min-pq.push( (new_state, getCost(new_state)) )
    else:
        return path_to(cur_state)
raise exception
```

Some simplifications are made here. In particular, any *state* mentioned are in fact *[Grid]* where the first element in *[Grid]* is the next state we are going to visit, and the last element in *[Grid]* is the *start state*.

3.4 Parallel Implementation Attempt #1

```
getNextNodes goal d st l xss = PQ.fromList $ zip costs neighbors where
costs = parMap rseq (getCost st d l goal) neighbors
neighbors = parMap rpar (:xss) (getNeighbors $ head xss)
```

For our initial attempt, we decided to spark the evaluation of the neighbors and costs in parallel using *rpar* and *rseq*. In addition, we also planned to start evaluation of all neighbors instead of adding them to a priority queue.

3.5 Parallel Implementation Attempt #2

For our second attempt, we decided to solve multiple N-Puzzles in parallel using different evaluation strategies including static partitioning and dynamic partitioning. To get a better evaluation on the performance parallel computation, we eliminated unnecessary sequential computation of our program. Only N-puzzle solutions' time complexities are printed at the end.

We started attempt #2 by changing *main.hs* file to make the solver solve multiple N-puzzles at the same time.

```
main :: IO ()
main = do
args <- checkArgs =<< getArgs
     content <- readFile $ args !! 0
     let   input_list = splitOn "#" content
           grids = map inputToGrid input_list
           solution = map solveNPuzzle grids
      print (length solution) >> mapM_ print solution
```

For the sequential evaluation approach, we simply called *map solveNPuzzle* on *grids*. Then, we implemented two parallel evaluation approaches. The first approach is static partitioning. We split *grids* to two lists *as'* and *bs'* and used *rpar* and *force* to solve N-puzzles.

```
solution = runEval $ do
                as' <- rpar (force (map solveNPuzzle as))
                bs' <- rpar (force (map solveNPuzzle bs))
                rseq as'
                rseq bs'
                return (as'++ bs')
```

When using static partitioning, we parallelly solve two lists of N-puzzles. We can only take advantage of two cores of CPU. To further evaluate parallel solving N-puzzles with more cores, we also implemented dynamic partitioning with *parList*.

```
parMap :: (a -> b) -> [a] -> Eval [b]
parMap _ [] = return []
parMap f (a:as) = do b <- rpar (f a)
                     bs <- parMap f as
```

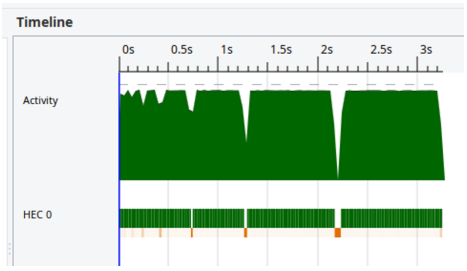
```
                    return (b:bs)
solution = runEval (parMap solveNPuzzle grids)
```

We used two set of N-puzzles to evaluate the two parallel approaches. The first set of N-puzzles is 1000 8-puzzles, and the second set is 40 15-puzzles. 8-puzzles are easy to solve, with most of the solutions' time complexities are within 1000. 15-puzzles are much harder, with some of the 15-puzzles' solutions exceeding 200000. We would like to find if the complexity of each puzzle can affect the program.

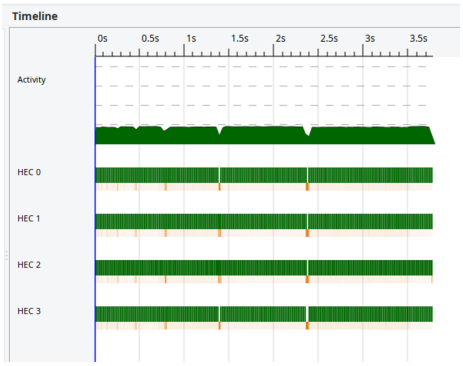
4. Performance Testing & Results

4.1 Results from Attempt #1

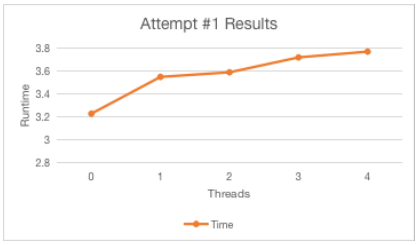
The initial attempt to construct *next state* and their cost in parallel only produced negative performance improvement across our tests. We believe that our strategy did not work out due to the limitation of setup of N-Puzzle. Since each puzzle only has a maximum of 4 *next state*s, constructing them in parallel adds too much overhead in comparison to the computation done. Additionally, getCost uses Manhattan Distance which requires very little calculation. We suspect if the cost function is more complicated, we might start seeing performance gains by evaluating them in parallel.



Without parallelization, done in 3.23s.



With parallelization and -N4, done in 3.77s.



Performance degradation as more threads are used

Our attempt to evaluate every *next state*s as soon as they are defined also failed. Due to the nature of the search algorithm, we needed to maintain a global state of visited states visible to all parallel evaluations so they don't repeat the same work. Because there is no easy way to accomplish that in Haskell, we decided to proceed without it. In practice, we found this modification brings no improvement over sequential evaluation for easy puzzles, again due to the overhead. We also found that this modification breaks the program when it is running on slightly more difficult puzzles, due to the amount of repeated work in parallel stalling the system.

4.2 Results from Attempt #2

The evaluation results of solving 1000 8-puzzles with different parallel approaches are as following.

Approach	Time	Speedup
sequential	6.438s	1x
static partitioning -N2	3.419s	1.88x
dynamic partitioning -N2	4.429s	1.45x
dynamic partitioning -N4	3.044s	2.11x
dynamic partitioning -N8	2.399s	2.68x

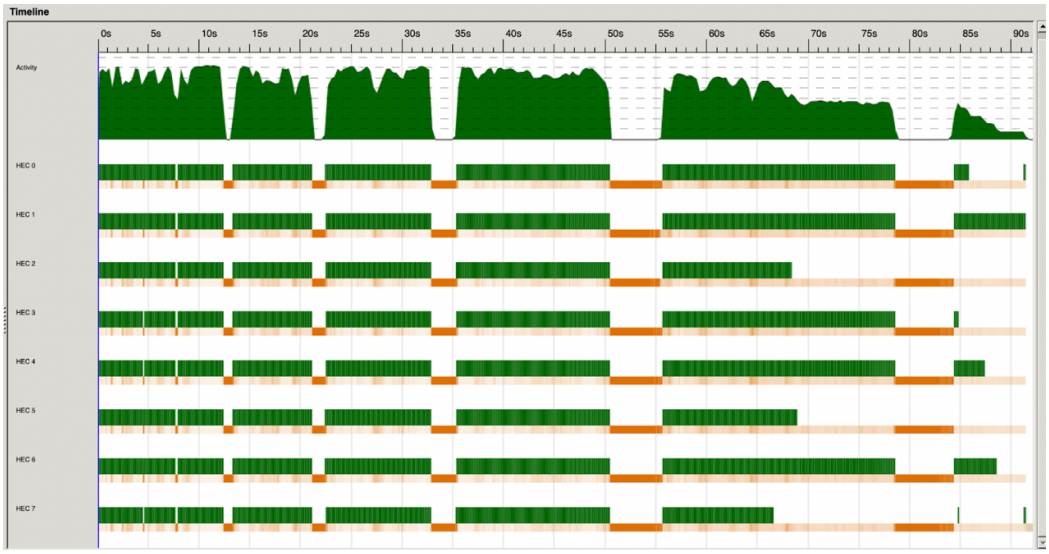
We could see the static partitioning approach has a pretty good performance with 2 cores. The speedup is 1.88x which is close to 2. By contrast, the dynamic partitioning approach with 2 cores doesn't perform comparably good. Its speedup is only 1.45x. When 4 cores and 8 cores are used, the speedup increase to 2.11x and 2.68x. It seems that if we want the best performance of solving multiple N-puzzles, we can just use more cores with dynamic partitioning. However, when evaluating the three approaches on solving 15-puzzles, we found it is not the case.

The evaluation results of solving 40 15-puzzles with different parallel approaches are as following.

Approach	Time	Speedup
sequential	177.366s	1x

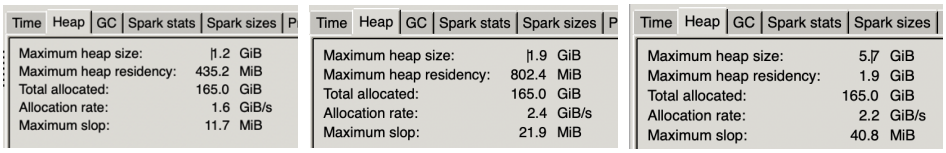
dynamic partitioning -N2	117.050s	1.52x
dynamic partitioning -N4	77.374s	2.92x
dynamic partitioning -N8	92.10s	1.93x

From the results, we could intuitively see that 15-puzzle is much harder to solve. The average time to solve one 15-puzzle is around 700x of the time to solve. When N-puzzles are much harder to solve, dynamic partitioning still can help accelerate the solver program. With 2 cores and 4 cores, the speedups are better than the speedups on solving 8-puzzles. However, when 8 cores are used, the performance declines a lot. It takes extra 15 seconds to finish solving 15-puzzles when using extra 4 cores. Thus, we took a look at the eventlog on threadscope.



eventlog when using 8 cores

On the eventlog, we could see the unusual patterns of several time intervals where the activity becomes zero and CPU cores are busy to deal with garbage collection. And along the program running, each time the pattern appears, the time interval become larger. At around 65 seconds, several cores start idling. Several intervals of garbage collection and several CPU cores idling after 65 seconds cause the program which utilize 8 cores runs slower than the program which utilize 4 cores.



heap sizes when using 2, 4, and 8 cores

We inspected some other metrics on eventlog and found when using 8 cores to solving 40 15-puzzles, the maximum heap size increases to 5.9 GiB which is 3 times

of the maximum heap size when using 4 cores. Large heap probably causes the slowdown.

The unusual slowdown of using 8 cores to solve 15-puzzles suggests us that to solve computation heavy problems, it is not always true that utilizing more resources can lead to a better result. We need to know that some unexpected things can emerge and can become counterproductive.

Code：

```
module Main where
import GHC.Conc(par)
import Control.Parallel.Strategies hiding(parMap)
import Control.DeepSeq
import Data.List.Split
import System.Environment (getArgs)
import System.Exit (exitSuccess)
import Grid (getSolvedGrid, Grid)
import Solver (solve, solve')
import Logger
import Parser

checkArgs :: [String] -> IO [String]
checkArgs xs = if null xs then displayHelp >> exitSuccess else pure xs

inputToGrid :: String -> [[Int]]
inputToGrid input = transformToGrid (words <$> (drop 1 . clearInput . lines
$ input))

solveNPuzzle :: Grid -> Int
solveNPuzzle grid = solve' (getSolvedGrid $ length grid) grid (Nothing, Nothing)

parMap :: (a -> b) -> [a] -> Eval [b]
parMap _ [] = return []
parMap f (a:as) = do b <- rpar (f a)
                     bs <- parMap f as
                     return (b:bs)

main :: IO ()
main = do
    args <- checkArgs =<< getArgs
    content <- readFile $ args !! 0
    let input_list = splitOn "#" content
        grids = map inputToGrid input_list
          {- map solveNPuzzle grids -}
```

```
     {- runEval (parMap solveNPuzzle grids) -}
        (as, bs) = splitAt (length grids `div` 2) grids
        solution = runEval $ do
                    as' <- rpar (force (map solveNPuzzle as))
                    bs' <- rpar (force (map solveNPuzzle bs))
                    rseq as'
                    rseq bs'
                    return (as'++ bs')
     print (length solution) >> mapM_ print solution

module Solver (SearchType(..), readSearchType, solve') where
    import qualified Data.PQueue.Prio.Min as PQ
    import qualified Data.HashSet as S
    import Logger
    import Grid
    import Distance

    data SearchType = Astar | Uniform | Greedy deriving Eq
    type NextNodesFunc = Int -> [Grid] -> PQ.MinPQueue Int [Grid]

    instance Show SearchType where
        show Astar      = "A*"
        show Uniform    = "Uniform cost"
        show Greedy     = "Greedy"

    readSearchType :: String -> Maybe SearchType
    readSearchType s = case s of
        "astar"     -> Just Astar
        "uniform"   -> Just Uniform
        "greedy"    -> Just Greedy
        _           -> Nothing

    -- Returns the cost of a node according to the SearchType currently used
    getCost :: SearchType -> Distance -> Int -> Grid -> Grid -> Int
    getCost st d l grid goal = let dist = calcDistance d grid goal in case st of
        Astar     -> dist + l  -- A* : h cost + g cost
        Uniform   -> l         -- Uniform : g cost only
        Greedy    -> dist      -- Greedy : h cost only

    -- Returns a PQueue containing the next nodes (value + cost)
    getNextNodes :: Grid -> Distance -> SearchType -> Int -> [Grid] -> PQ.MinPQueue
Int [Grid]
    getNextNodes goal d st l xss = PQ.fromList $ zip costs neighbors where
        costs       = (getCost st d l goal) <$> head <$> neighbors
```

```
        neighbors   = map (:xss) $ getNeighbors $ head xss

-- goal : stage to reach ; xss : path from begining to current node ; os : open
set ; cs : close set ; nn : nextNodes function ; n : time complexity ; m : space
complexity ; l : xss length
    runSearch' :: Grid -> [Grid] -> PQ.MinPQueue Int [Grid] -> S.Set Grid ->
NextNodesFunc -> Int -> Int -> Int -> Int
    runSearch' goal xss os cs nn n m l
        | head xss == goal                          = n
        | suc /= PQ.empty                           = runSearch' goal  ((minim
suc):xss)  (PQ.union os $ PQ.deleteMin suc)  cs'  nn  (n+1)  size (l+1)
        | suc == PQ.empty && os /= PQ.empty         = runSearch' goal  (tail xss)
os                                cs'  nn  (n+1)  size (l-1)
        | otherwise = 0 where
            suc     = PQ.filter (\x -> S.notMember (head x) cs) $ nn l xss
            cs'     = S.insert (head xss) cs
            minim x = head . snd $ PQ.findMin x
            size    = if PQ.size os > m then PQ.size os else m

solve' :: Grid -> Grid -> (Maybe SearchType, Maybe Distance) -> Int
    solve' goal xs (Nothing, Nothing) = runSearch' goal [xs]  PQ.empty  S.empty
( getNextNodes goal defaultHeuristic defaultSearch )  0 0 1
    solve' goal xs (Just st, Nothing) = runSearch' goal [xs]  PQ.empty  S.empty
( getNextNodes goal defaultHeuristic st            )  0 0 1
    solve' goal xs (Nothing, Just d)  = runSearch' goal [xs]  PQ.empty  S.empty
( getNextNodes goal d                defaultSearch )  0 0 1
    solve' goal xs (Just st, Just d)  = runSearch' goal [xs]  PQ.empty  S.empty
( getNextNodes goal d                st            )  0 0 1


    import Distance
        import Solver

        maxPuzzle :: Int
        maxPuzzle = 999

        -- Strips comments and empty lines
        clearInput :: [String] -> [String]
        clearInput xs = filter (/="") $ map (head . splitOn "#") xs

        -- Checking size of the puzzle
        isValidSize :: [[Int]] -> Bool
        isValidSize xss = let ys = map (^2) [3..maxPuzzle] in (length . concat) xss
`elem` ys
```

```
        -- Sorts input and compare it to an enum list of the same size
        hasValidContent :: [[Int]] -> Bool
        hasValidContent xss = let n = length xs - 1; xs = concat xss; ys = [0..n]
in sort xs == ys

        -- Returns the input as [[Int]] if it is valid, otherwise returns Nothing
        transformInput :: [[String]] -> Maybe [[Int]]
        transformInput xss
            | all isDigit (concat $ concat xss) == False = Nothing
            | otherwise = let xss' = (map read) <$> xss in if isValidSize xss' &&
hasValidContent xss' then Just xss' else Nothing

        transformToGrid :: [[String]] -> [[Int]]
        transformToGrid xss =
            let xss' = (map read) <$> xss in xss'

        -- Check programs args, and returns associated flags
        parseArgs :: [String] -> (Maybe SearchType, Maybe Distance)
        parseArgs xs = case length xs of
            1   -> (Nothing, Nothing)
            2   -> (readSearchType (xs !! 1), Nothing)
            _   -> (readSearchType (xs !! 1), readDistance (xs !! 2))
```