Question 1

Input: a sentence $s = x_1 \dots x_n$, a context-free grammar $G = (N, \Sigma, S, R)$.

Initialization:

For all $i \in \{1 \dots n\}$, for all $X \in N$,

$$\pi(i, i, X) = \begin{cases} 1 & \text{if } X \rightarrow x_i \in R \\ 0 & \text{otherwise} \end{cases}$$

Algorithm:

- For $l = 1 \dots (n-1)$
  - For $i = 1 \dots (n-l)$
    - Set $j = i + l$
    - For all $X \in N$, calculate

$$\pi(i, j, X) = \sum_{\substack{X \rightarrow Y\,Z \in R, \\ s \in \{i \dots (j-1)\}}} \pi(i, s, Y) \times \pi(s+1, j, Z)$$

Output: Return $\pi(1, n, S)$

Question 2

Base case: for all $i = 1 \ldots n$, for all $X \in N$,

$$\pi(i, i, X) = \begin{cases} q(X \rightarrow x_i) & \text{if } X \rightarrow x_i \in R \\ 0 & \text{otherwise} \end{cases}$$

Recursive case:

- For $l = 1 \ldots (n-1)$
  - Set $j = 1 + l$
  - For all $X \in N$, calculate

$$\begin{aligned} & \pi(1, j, X) \\ = & \max_{X \rightarrow YZ \in R} \left(q(X \rightarrow YZ) \times \pi(1, j-1, Y) \times \pi(j, j, Z) \right) \end{aligned}$$

Output: Return $\pi(1, n, S) = \max_{t \in \mathcal{T}(s)} p(t)$

Question 3 (Simple solution: but rule probabilities don't sum to one)

S $\rightarrow$ A FA	$q(A\|*)$
S $\rightarrow$ B FB	$q(B\|*)$
S $\rightarrow$ A	$q(A\|*) \times q(STOP\|A)$
S $\rightarrow$ B	$q(B\|*) \times q(STOP\|B)$
FA $\rightarrow$ A FA	$q(A\|A)$
FA $\rightarrow$ A	$q(A\|A) \times q(STOP\|A)$
FA $\rightarrow$ B FB	$q(B\|A)$
FA $\rightarrow$ B	$q(B\|A) \times q(STOP\|B)$
FB $\rightarrow$ A FA	$q(A\|B)$
FB $\rightarrow$ A	$q(A\|B) \times q(STOP\|A)$
FB $\rightarrow$ B FB	$q(B\|B)$
FB $\rightarrow$ B	$q(B\|B) \times q(STOP\|B)$
A $\rightarrow$ s	$e(s\|A)$
A $\rightarrow$ t	$e(t\|A)$
B $\rightarrow$ s	$e(s\|B)$
B $\rightarrow$ t	$e(t\|B)$

Question 3 (with rule probabilities summing to one)

Note: for any X, Y define $q'(X|Y) = \frac{q(X|Y)}{1-q(STOP|Y)}$

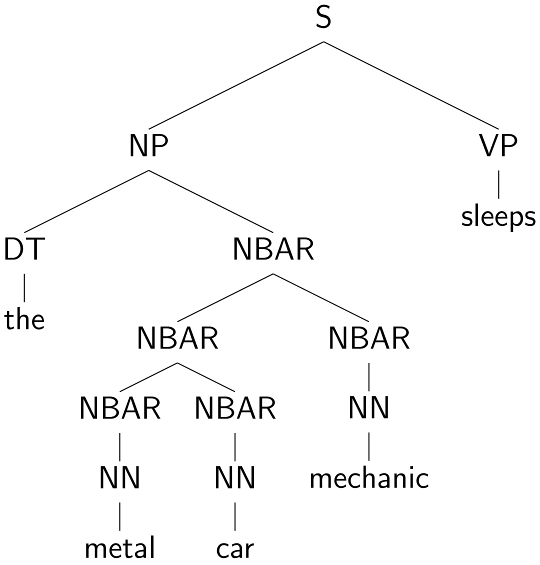
S $\rightarrow$ A FA	$q(A|*) \times (1 - q(STOP|A))$
S $\rightarrow$ B FB	$q(B|*) \times (1 - q(STOP|B))$
S $\rightarrow$ A	$q(A|*) \times q(STOP|A)$
S $\rightarrow$ B	$q(B|*) \times q(STOP|B)$
FA $\rightarrow$ A FA	$q'(A|A) \times (1 - q(STOP|A))$
FA $\rightarrow$ A	$q'(A|A) \times q(STOP|A)$
FA $\rightarrow$ B FB	$q'(B|A) \times (1 - q(STOP|B))$
FA $\rightarrow$ B	$q'(B|A) \times q(STOP|B)$
FB $\rightarrow$ A FA	$q'(A|B) \times (1 - q(STOP|A))$
FB $\rightarrow$ A	$q'(A|B) \times q(STOP|A)$
FB $\rightarrow$ B FB	$q'(B|B) \times (1 - q(STOP|B))$
FB $\rightarrow$ B	$q'(B|B) \times q(STOP|B)$
A $\rightarrow$ s	$e(s|A)$
A $\rightarrow$ t	$e(t|A)$
B $\rightarrow$ s	$e(s|B)$
B $\rightarrow$ t	$e(t|B)$

Question 4

All parse trees for this sentence contain the following rules:

S $\rightarrow$ NP VP
NP $\rightarrow$ DT NBAR
NBAR $\rightarrow$ NN (three times)
NBAR $\rightarrow$ NBAR NBAR (twice)
VP $\rightarrow$ sleeps
DT $\rightarrow$ the
NN $\rightarrow$ mechanic
NN $\rightarrow$ car
NN $\rightarrow$ metal

Because all parse trees contain the same set of rules, the probabilities for the different parse trees are all identical.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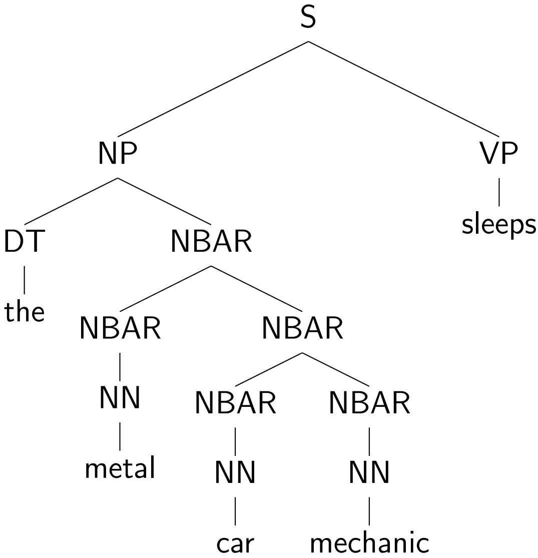
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