

# **Foundations of Graphical Models**

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Columbia University

## Today's lecture

- What is this course about?
- Latent Dirichlet allocation: An example of a graphical model
- Other examples of applied probabilistic modeling
- Box's loop
- What will we cover?
- Prerequisites, requirements, and grades

What is this course about?

# Latent Dirichlet Allocation

(An example of a model that I know well)



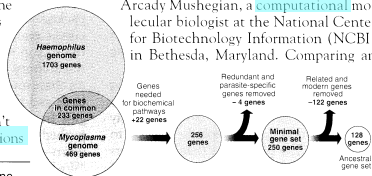
## Seeking Life's Bare (Genetic) Necessities

COLD SPRING HARBOR, NEW YORK—How many genes does an organism need to survive? Last week at the genome meeting here,\* two genome researchers with radically different approaches presented complementary views of the basic genes needed for life. One research team, using computer analyses to compare known genomes, concluded that today's organisms can be sustained with just 250 genes, and that the earliest life forms required a mere 128 genes. The other researcher mapped genes in a simple parasite and estimated that for this organism, 800 genes are plenty to do the job—but that anything short of 100 wouldn't be enough.

Although the numbers don't match precisely, those predictions

“are not all that far apart,” especially in comparison to the 75,000 genes in the human genome, notes Siv Andersson of Uppsala University in Sweden, who arrived at the 800 number. But coming up with a consensus answer may be more than just a genetic numbers game, particularly as more and more genomes are completely mapped and sequenced. “It may be a way of organizing any newly sequenced genome,” explains

Arcady Mushegian, a computational molecular biologist at the National Center for Biotechnology Information (NCBI) in Bethesda, Maryland. Comparing an



Stripping down. Computer analysis yields an estimate of the minimum modern and ancient genomes.

\* Genome Mapping and Sequencing, Cold Spring Harbor, New York, May 8 to 12.

Documents exhibit multiple topics.

## Topics

gene 0.04  
dna 0.02  
genetic 0.01  
...

life 0.02  
evolve 0.01  
organism 0.01  
...

brain 0.04  
neuron 0.02  
nerve 0.01  
...

data 0.02  
number 0.02  
computer 0.01  
...

## Documents

### Seeking Life's Bare (Genetic) Necessities

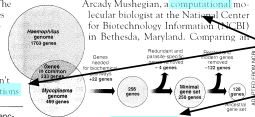
COLD SPRING HARBOR, NEW YORK—How many **genes** does an **organism** need to **survive**? Last week at the genome meeting here, two genome researchers with radically different approaches presented complementary views of the basic genes needed for **life**. One research team, using **computer** analyses to compare known **genomes**, concluded that today's **organisms** can be sustained with just 250 **genes**, and that the earliest life forms required a mere 128 **genes**. The other researcher mapped genes in a simple parasite and estimated that for this organism, 800 genes are plenty to do the job—but that anything short of 100 wouldn't be enough.

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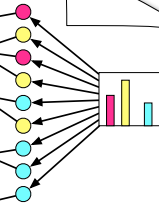
SCIENCE • VOL. 272 • 24 MAY 1996

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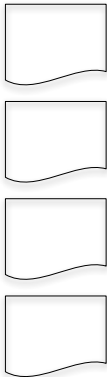
**Stripping down.** Computer analysis yields an estimate of the minimum modern and ancient genomes.

## Topic proportions and assignments



## Latent Dirichlet Allocation

## Topics



## Documents

## Topic proportions and assignments

### Seeking Life's Bare (Genetic) Necessities

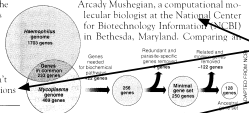
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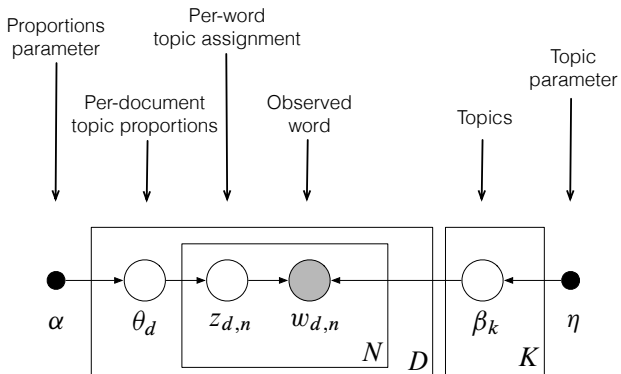
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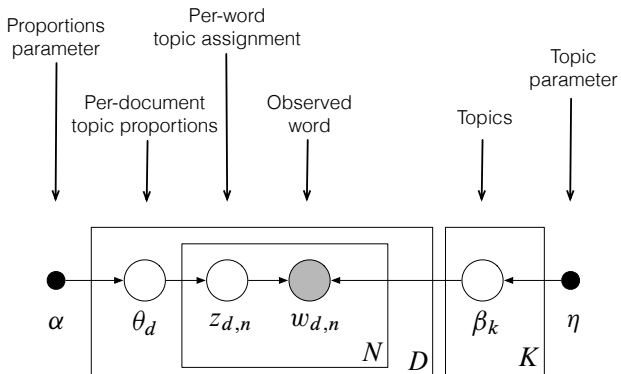
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## Latent Dirichlet Allocation



### LDA as a graphical model

- Nodes are random variables; edges indicate dependence.
- Shaded nodes are observed; unshaded nodes are hidden.
- Plates indicate replicated variables.



### LDA as a graphical model

- Encodes independence assumptions
- Defines a factorization of the joint distribution
- Connects to algorithms for computing with data





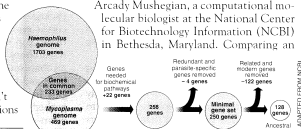
- **Data:** The OCR'ed collection of *Science* from 1990–2000
  - 17K documents
  - 11M words
  - 20K unique terms (stop words and rare words removed)
- **Model:** 100-topic LDA model using variational inference.

## Seeking Life's Bare (Genetic) Necessities

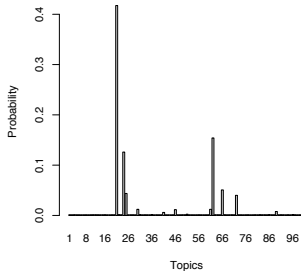
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human	evolution	disease	computer
genome	evolutionary	host	models
dna	species	bacteria	information
genetic	organisms	diseases	data
genes	life	resistance	computers
sequence	origin	bacterial	system
gene	biology	new	network
molecular	groups	strains	systems
sequencing	phylogenetic	control	model
map	living	infectious	parallel
information	diversity	malaria	methods
genetics	group	parasite	networks
mapping	new	parasites	software
project	two	united	new
sequences	common	tuberculosis	simulations

perspective identifying tumor suppressor genes in human...  
letters global warming report leslie roberts article global...  
research news a small revolution gets under way the 1990s....  
a continuing series the reign of trial and error draws to a close...  
making deep earthquakes in the laboratory lab experimenters...  
quick fix for freeways thanks to a team of fast working...  
feathers fly in grouse population dispute researchers...

....

245 1897:1 1467:1 1351:1 731:2 800:5 682:1 315:6 3668:1 14:1  
260 4261:2 518:1 271:6 2734:1 2662:1 2432:1 683:2 1631:7  
279 2724:1 107:3 518:1 141:3 3208:1 32:1 2444:1 182:1 250:1  
266 2552:1 1993:1 116:1 539:1 1630:1 855:1 1422:1 182:3 2432:1  
233 1372:1 1351:1 261:1 501:1 1938:1 32:1 14:1 4067:1 98:2  
148 4384:1 1339:1 32:1 4107:1 2300:1 229:1 529:1 521:1 2231:1  
193 569:1 3617:1 3781:2 14:1 98:1 3596:1 3037:1 1482:12 665:2

....

```
docs <- read.documents("mult.dat")
K <- 20
alpha <- 1/20
eta <- 0.001
model <- lda.collapsed.gibbs.sampler(documents, K, vocab, 1000, alpha, eta)
```

## LDA in R

1 dna gene sequence genes sequences human genome genetic analysis two	2 protein cell cells proteins receptor fig binding activity activation kinase	3 water climate atmospheric temperature global surface ocean carbon atmosphere changes	4 says researchers new university just science like work first years	5 mantle high earth pressure seismic crust temperature earths lower earthquakes
6 end article start science readers service news card circle letters	7 time data two model fig system number element math --	8 materials surface high structure temperature molecules chemical molecular fig university	9 dna rna transcription protein site binding sequence proteins specific sequences	10 disease cancer patients human gene medical studies drug normal drugs
11 years million ago age university north early fig evidence record	12 protein evolution population evolutionary university populations natural studies genetic biology	13 protein structure proteins two amino binding acid residues molecular structural	14 cells cell virus hiv infection immune human antigen infected viral	15 space solar observations earth stars university mass sun astronomers telescope
16 fax manager science aaas advertising sales member recruitment associate washington	17 cells cell gene genes expression development mutant mice fig biology	18 energy electron state light quantum physics electrons high laser magnetic	19 research science national scientific scientists new states university united health	20 neurons brain cells activity fig channels university cortex neuronal visual

## Wikipedia Topics

Relative Presence of Topics In all Documents

{household, population, female}

{film, series, show}

{theory, work, human}

{son, year, death}

{war, force, army}

{system, computer, user}

{album, band, music}

{government, party, election}

{game, team, player}

{god, call, give}

{company, market, business}

{math, number, function}

{law, state, case}

## {film, series, show}

words	related documents	related topics
film	The X-Files	{son, year, death}
series	Orson Welles	{work, book, publish}
show	Stanley Kubrick	{album, band, music}
character	B movie	{woman, child, man}
play	Mystery Science Theater 3000	{law, state, case}
make	Monty Python	{black, white, people}
episode	Doctor Who	{theory, work, human}
movie	Sam Peckinpah	{{@card@}, make, design}
good	Married... with Children	{war, force, army}
release	History of film	{god, call, give}
feature	The A-Team	{game, team, player}
television	Pulp Fiction (film)	{day, year, event}
star	Mad (magazine)	{company, market, business}

## Stanley Kubrick



### related topics

{film, series, show}  
 {theory, work, human}  
 {son, year, death}  
 {black, white, people}  
 {god, call, give}  
 {math, energy, light}

**Stanley Kubrick** (July 26, 1928 – March 7, 1999) was an American film director, writer, producer, and photographer who lived in England during most of the last four decades of his career. Kubrick was noted for the scrupulous care with which he chose his subjects, his slow method of working, the variety of genres he worked in, his technical perfectionism, and his reticence about his films and personal life. He worked far beyond the confines of the Hollywood system, maintaining almost complete artistic control and making movies according to his own whims and time constraints, but with the rare advantage of big-studio financial support for all his endeavors.

Kubrick's films are characterized by a formal visual style and meticulous attention to detail—his later films often have elements of surrealism and expressionism that eschews structured linear narrative. His films are repeatedly described as slow and methodical, and are often perceived as a reflection of his obsessive and perfectionist nature.<sup>[1]</sup> A recurring theme in his films is man's inhumanity to man. While often viewed as

### related documents

Orson Welles  
 B movie  
 Mystery Science Theater 3000  
 Monty Python  
 Doctor Who  
 Sam Peckinpah  
 The A-Team  
 Pulp Fiction (film)  
 Buffy the Vampire Slayer (TV series)  
 The X-Files  
 Sunset Boulevard (film)  
 Jack Benny

## {theory, work, human}

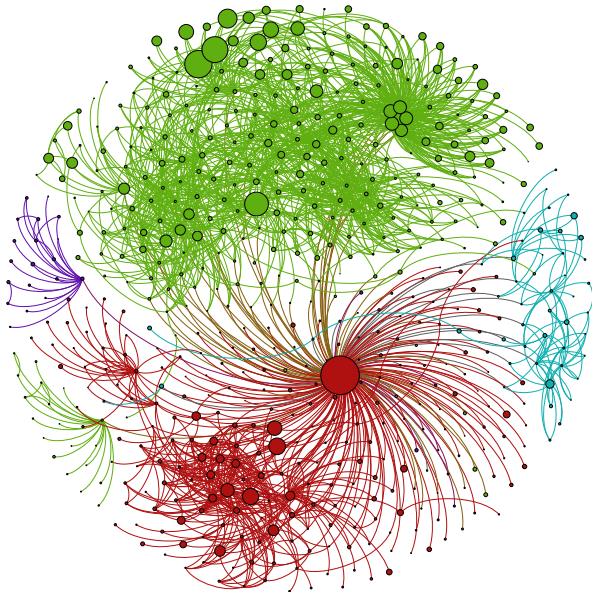
words	related documents	related topics
theory	Meme	{work, book, publish}
work	Intelligent design	{law, state, case}
human	Immanuel Kant	{son, year, death}
idea	Philosophy of mathematics	{woman, child, man}
term	History of science	{god, call, give}
study	Free will	{black, white, people}
view	Truth	{film, series, show}
science	Psychoanalysis	{war, force, army}
concept	Charles Peirce	{language, word, form}
form	Existentialism	{{@card@}, make, design}
world	Deconstruction	{church, century, christian}
argue	Social sciences	{rate, high, increase}
social	Idealism	{company, market, business}



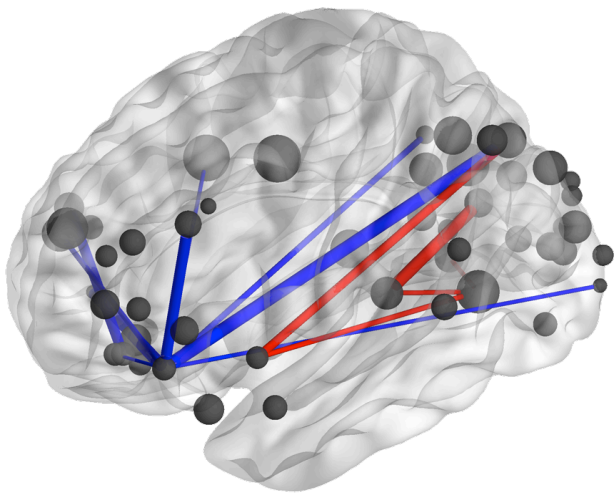
Topics found in 1.8M articles from the New York Times

## Other examples of applied probabilistic modeling

(from my research group and others)

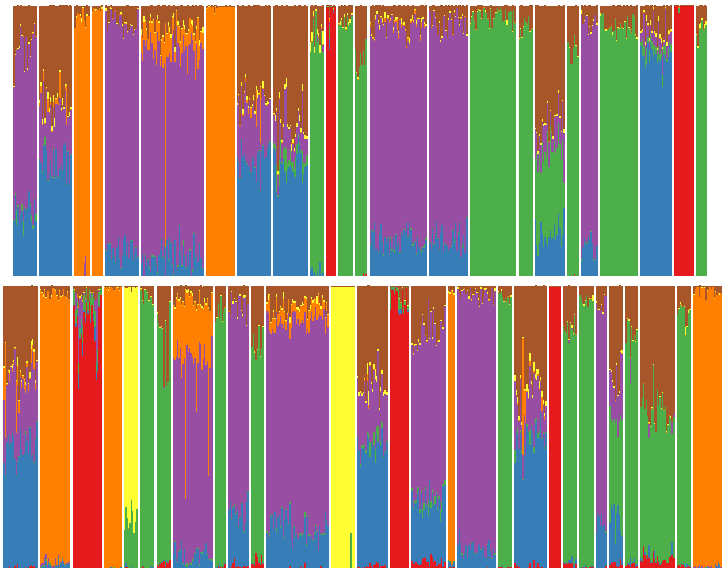


Communities discovered in a 3.7M node network of U.S. Patents

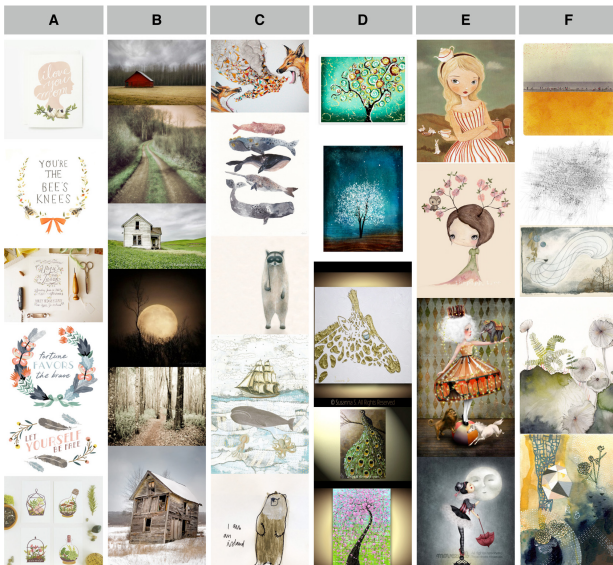


Neuroscience analysis of 220 million fMRI measurements





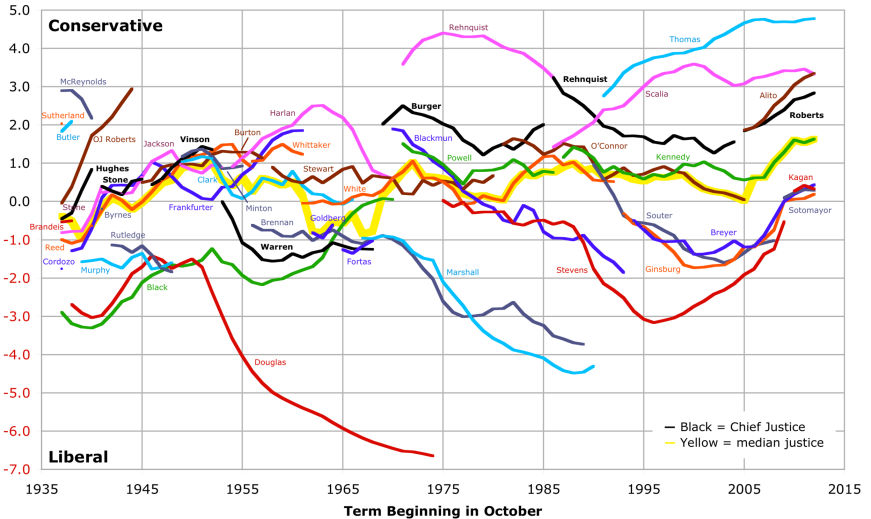
Population analysis of 2 billion genetic measurements



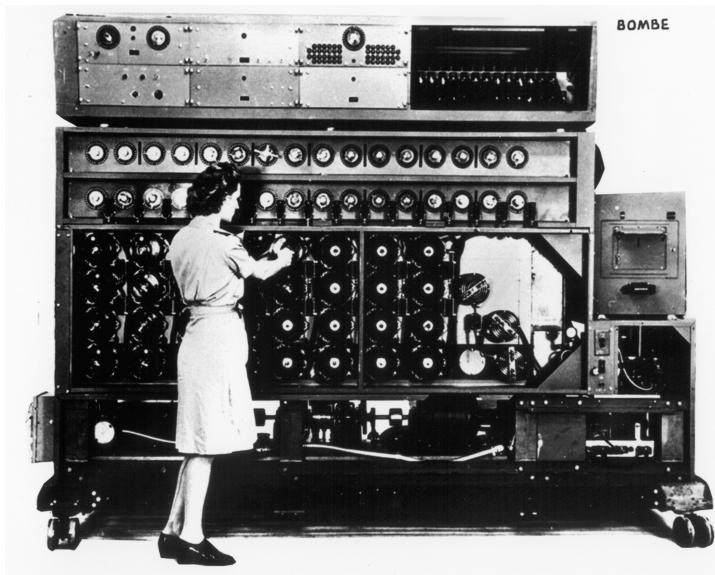
Patterns of preferences found at Etsy.com (Hu et al., 2014)

# Ideological Leanings of Supreme Court Justices

Source Data: Andrew D. Martin and Kevin M. Quinn  
<http://mqscores.wustl.edu/measures.php>

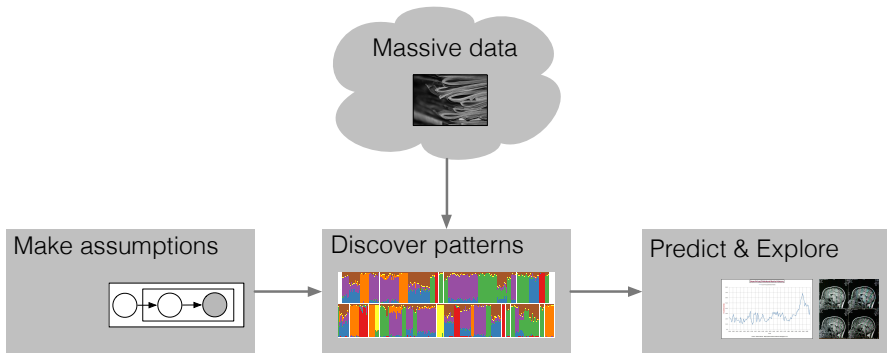


Supreme Court Ideology over time (Martin and Quinn, 2001)



Breaking the Nazi code (Turing and Good, 194?)

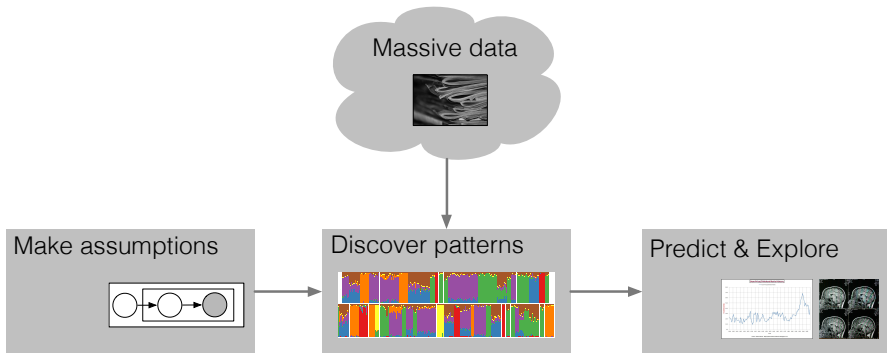
# Box's Loop



## Why we like this picture:

- Customized data analysis is important to many fields.
- This pipeline separates assumptions, computation, application.
- It facilitates solving data science problems.

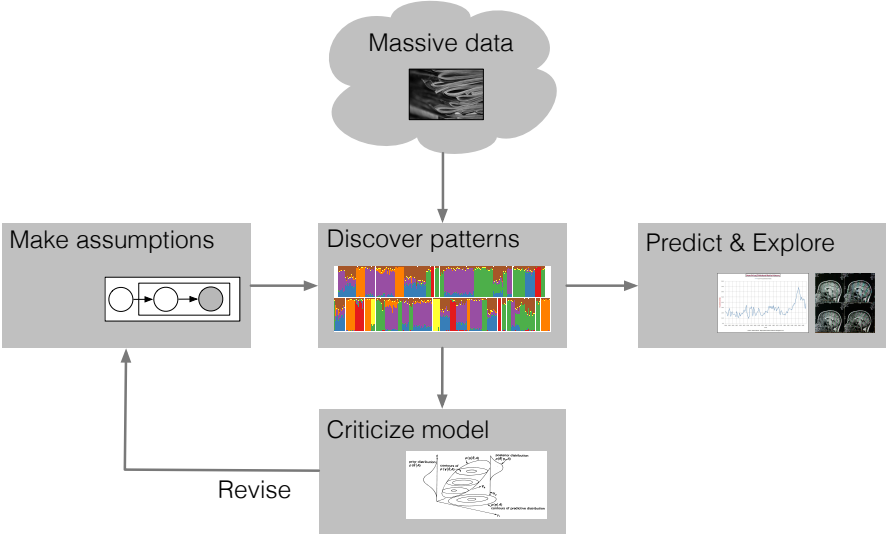
# Box's Loop



## What we need:

- Expressive components from which to build models
- Scalable and generic inference algorithms
- Stretch probabilistic modeling into new areas

# Box's Loop



What will we cover?



# The basics of graphical models

1. Probability: Basic concepts and review
2. Semantics of graphical models
3. D-separation and conditional independence
4. The elimination algorithm
5. Tree propagation and hidden Markov models I
6. Tree propagation and hidden Markov models II

## Latent variable models

1. Models, data, and statistical concepts I
2. Models, data, and statistical concepts II
3. Bayesian mixtures of Gaussians and the Gibbs sampler I
4. Bayesian mixtures of Gaussians and the Gibbs sampler II
5. Exponential families, conjugacy, and mixtures of exponential families I
6. Exponential families, conjugacy, and mixtures of exponential families II
7. Mixed-membership, topic models, and variational inference I
8. Mixed-membership, topic models, and variational inference II
9. Matrix factorization and recommendation systems I
10. Matrix factorization and recommendation systems II

## Conditional models

1. Regression: Linear and logistic
2. Generalized linear models
3. Hierarchical models, robust models, and empirical Bayes I
4. Hierarchical models, robust models, and empirical Bayes II

## Advanced ideas in approximate posterior inference

1. Markov chain Monte Carlo I
2. Markov chain Monte Carlo II
3. Variational inference I
4. Variational inference II

## Other topics and summary

1. An brief introduction to Bayesian nonparametrics
2. Summary (and wiggle room)

## Some additional discussion

- Programming languages
- Applications
- Box's loop, again
- Note: We will usually be at the board.

## Prerequisites, Requirements, Grades, Etc.

- <http://www.cs.columbia.edu/~blei/fogm/>
- Office hours: Wednesdays 2:30-4:30, 703 CEPSR
- Prerequisites
  - Probability and Statistics
  - Optimization
  - Programming
- Requirements
  - Weekly paper about the reading (< 1 page)
  - Occasional homework
  - Final project
- Your grade: Mostly the final project