COMS W3261 CS Theory: Homework 5. Assigned Nov 29, 2017.
Answers in PDF due by 11:59pm Dec 11, 2017
on Courseworks/COMSW3261/Assignments.

Each problem is worth 20 points. You can discuss problems with others but your answers must be in your own words. Late assignments cannot be accepted.

1. What would happen if someone discovered an NP-complete language L that was in P? Justify your answer.

2. Suppose we know that there is a polynomial-time reduction of a language L to SAT. What can we say about L? Justify your answer.

3. The game PEBBLES is played on a $k \times n$ chessboard. Initially each square of the chessboard has a black pebble, or a white pebble, or no pebble. You play the game by removing pebbles one at a time. You win the game if you can end up with a board in which each column contains only pebbles of a single color and each row contains at least one pebble.

   (a) Show that the set of winnable PEBBLES games is in NP by describing a nondeterministic polynomial-time algorithm to determine whether a given PEBBLES board is winnable.

   (b) Given a boolean expression E in 3-CNF with k clauses and n variables, construct the following $k \times n$ board: If literal x_i is in clause c_j, put a black pebble in column x_i, row c_j. If literal $\neg x_i$ is in clause c_j, put a white pebble in column x_i, row c_j. Show that E is satisfiable if and only if this PEBBLES game is winnable. [See HMU, Section 10.3.1, p. 448, for a definition of 3-CNF.]

   (c) What can you conclude from (a) and (b)?

4. PAC-learning problem. Suppose we have a collection of 100 concepts. How many samples do we need to examine to find a true concept with an error of at most 0.1 and a probability of at least 95%?

5. Consider the lambda expression $(\lambda x.\, a\, x)((\lambda y.\, b\, y)\, c)$.

   (a) Identify all redexes in this expression.

   (b) Evaluate this expression using normal order evaluation.

   (c) Evaluate this expression using applicative order evaluation.

6. Let G be the function definition $(\lambda f.\, \lambda x.\, f(f\, x))$. Evaluate the lambda expression GG.

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