Sample Midterm COMS W3261 CS Theory, Section 1
October 23, 2017; 75 minutes

Instructions

- Problems 1-5 are each worth 20 points. Problem 6 is extra credit (10 points) and optional. No aids permitted.

Problems

1. Consider the grammar G with the productions $S \to aSa \,|\, bSb \,|\, aa$.
   (a) Describe in words the language L generated by this grammar.
   (b) Using induction both ways, prove that $L(G) = L$.
2. Consider the grammar G in problem 1.
   (a) Transform G into an equivalent Chomsky-Normal-Form grammar G'.
   (b) Construct a Cocke-Younger-Kasami parsing table for G' and the sentence $baab$.
   (c) Show how to reconstruct a parse tree for the sentence $baab$ from the CYK table.
3. Using the pumping lemma for regular languages, prove that the language generated by the grammar in problem 1 is not regular.
4. Consider the regular expression $R = ab^*a$.
   (a) Construct the McNaughton-Yamada-Thompson ϵ-NFA for R.
   (b) Using the subset construction convert this ϵ-NFA into a DFA.
   (c) Minimize the number of states in your DFA.
   (d) Prove your DFA is minimum state.
5. Consider the language $L_1 - L_2$, i.e., the difference of the two languages L_1 and L_2.
   (a) If L_1 is regular and L_2 is context free, must $L_1 - L_2$ be regular? Briefly justify your answer.
   (b) If L_1 is context free and L_2 is regular, must $L_1 - L_2$ be context free? Briefly justify your answer.
6. **Extra Credit, 10 points.** Define $min(L) = \{w \,|\, w$ is in L but no proper prefix of w is in $L\}$. If L is a context-free language, is $min(L)$ always context free? Informally prove your answer.