

COMS W3261 CS Theory: Homework 2. Assigned Oct 11, 2017.  
Answers in PDF due Oct 19, 2017 on Courseworks/COMSW3261/Assignments.

Each problem is worth 20 points. You can discuss problems with others but your answers must be in your own words. Late assignments cannot be accepted.

1. State whether each of the following languages is context free or not. If it is, give a CFG for it. If it is not, briefly justify why it is not.
  - (a)  $L_1 = \{x \mid x \text{ is a postfix arithmetic expression with the operators } - \text{ and } / \text{ and the constant } \mathbf{num}\}$ . E.g.,  $1\ 2\ -\ 3\ /$  is a postfix arithmetic expression.
  - (b)  $L_2 = \{xx^R x \mid x \text{ is a string of } a\text{'s and } b\text{'s}\}$ .  $x^R$  is the reversal of the string  $x$ .
  - (c)  $L_3 = \{x \mid x \text{ is a string of } a\text{'s and } b\text{'s with more } a\text{'s than } b\text{'s}\}$ .
  - (d)  $L_4 = \{a^p \mid p \text{ is a prime number}\}$ .
2. Consider the grammar  $G$  with the productions  $S \rightarrow SaSb \mid SbSa \mid \epsilon$ .
  - (a) Describe  $L(G)$  in English.
  - (b) Prove  $G$  generates this language.
  - (c) Prove that  $L(G)$  is not regular.
  - (d) Show that  $G$  is ambiguous.
3. Put the grammar  $G$  in question (2) into Chomsky Normal Form. Use the Cocke-Younger-Kasami algorithm to parse the sentence  $abab$  according to your CNF grammar. Show how all the parse trees for the sentence  $abab$  can be constructed from the CYK table.
4. Using the pumping lemma for context-free languages, show that a proof that the language  $\{ww \mid w \text{ is a string of } a\text{'s and } b\text{'s}\}$  is not context free can be framed as a five-step adversarial game: (1) we pick, (2) adversary picks, (3) we pick, (4) adversary picks, (5) we win by making a winning pick.
5. For two languages  $L$  and  $M$ , let  $insert(L, M) = \{xyz \mid xz \text{ is in } L \text{ and } y \text{ is in } M\}$ . If  $L$  and  $M$  are context-free languages, is  $insert(L, M)$  always context free? Briefly justify your answer.
6. Consider the language  $L$  consisting of all strings of  $a$ 's and  $b$ 's that are even-length palindromes with the same number of  $a$ 's as  $b$ 's. If  $L$  is context free, construct a CFG for  $L$  and prove your grammar generates  $L$ . If  $L$  is not context free, prove that it is not.

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